## ТРАНСПОРТНЫЕ СИСТЕМЫ

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# MODIFICATION OF CLARK AND WRIGHT VEHICLE ROUTING PROBLEM ALGORYTHM

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**Abstract.** Based on the analysis of shortcomings of existing algorithms for solving the problem conveying the developed method modification of the method of Clarke and Wright is based on the dynamic transformation of the payoff matrix. To verify the effectiveness of the modified algorithm, the data of real beneficiaries of a batch of goods in the central part of Kharkiv was used as a test example. The results of the comparison showed that the mileage of vehicles on routes built using the modified algorithm is by about 0, 1-3, 8% less.

**Key words:** customer transport service, delivery route, route time, VRP, dynamic savings matrix, routes merging, savings account means, vehicle carrying capacity.

## МОДИФИКАЦИЯ АЛГОРИТМА КЛАРКА И РАЙТА МАРШРУТИЗАЦИИ ПЕРЕВОЗОК ПАРТИОННЫХ ГРУЗОВ

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Аннотация. Описан способ модификации метода Кларка и Райта, основанный на динамической трансформации матрицы выигрышей. Приведены результаты сравнения оригинального и модифицированного алгоритмов. В качестве тестового примера использованы данные 70 реальных получателей партионных грузов в центральной части г. Харькова. Результаты сравнения показали эффективность предложенной модификации алгоритма.

**Ключевые слова:** партионные грузы, задача развозки, матрица выигрышей, слияние маршрутов, вместимость автомобиля, способ расчета выигрышей.

# МОДИФІКАЦІЯ АЛГОРИТМУ КЛАРКА І РАЙТА МАРШРУТИЗАЦІЇ ПЕРЕВЕЗЕНЬ ГУРТОВИХ ВАНТАЖІВ

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**Анотація.** Описано спосіб модифікації методу Кларка і Райта, оснований на динамічній трансформації матриці виграшів. Наведено результати порівняння оригінального та модифікованого алгоритмів. Як тестовий приклад використано дані 70 реальних одержувачів гуртових вантажів у центральній частині м. Харкова. Результати порівняння показали ефективність запропонованої модифікації алгоритму.

**Ключові слова:** гуртові вантажі, завдання розвезення, матриця виграшів, злиття маршрутів, місткість автомобіля, спосіб розрахунку виграшів.

#### Introduction

In modern conditions, the requirements for increasing the economic efficiency of various

branches of the national economy are equally applicable to the road transport. The most costly in the road transport is the transportation of small lot items (which are cargoes whose shipment size is much less than the capacity of the car used to transport them), which is 45 % of the total range of goods [1]. At the same time, their share in the total traffic volume is only 2 %, and the share of transportation costs, due to the peculiarities of the transportation of batch cargo, is 32 % [2]. Such peculiarities of consignment shipments include: small shipments, a large number of recipients at one (rarely - more than one) sender, the presence of a difference (often significant) in the sizes of shipments, both between different consignees for one-day transportation, and one consignee for transportation on different days. In these conditions, one of the most effective ways to reduce the cost of transportation is to cut the mileage of cars in meeting all the requirements for transportation. Thus, the development of effective methods of routing the transportation of small lot items is an actual scientific and practical task.

## **Analysis of publications**

Clark and Wright developed a method for solving the problem of transportation more than half a century ago [3], however, the idea embedded in it proved to be so productive that the method in question became the most popular one. This method refers to heuristic methods and, accordingly, does not guarantee obtaining an optimal solution, and therefore attempts to increase the accuracy of solutions obtained with its help due to various modifications are still being made. The above modifications can be conditionally divided into those that are oriented to improve the accuracy of the algorithm under specific transportation conditions [4, 5] and such that have a general nature [6, 7].

Immediately after the appearance of the Clark-Wright method, several algorithms for local optimization of the routes constructed using this method were proposed, the most common of which was the inversion method [8] (n-optimal procedures originally used in the version of the two-optimal procedure, later, with a wide application of computer aids to solve the problem of transportation, have found further practice). The method of local optimization of Ren and Holiday [9] is also quite widespread. The development of ideas of Clark and Wright continues to this day, which is explained by the wide variety of transportation tasks setting. In addition, the solutions obtained with this method are basic for comparing both the efficiency of its modifications and other algorithms [10].

## Formulation of the problem

The task of transportation is a kind of traveling salesman's problem, in which the capacity of a «backpack» of a traveling salesman is less than the total volume of goods delivery to all «cities». The mathematical formulation of the traveling salesman problem is given in [4]. The difference between the formulation in question and other statements of transportation tasks is as follows: 1 – the turn-around time of the vehicle on the route is limited; 2 – the service time of the check-in points on the route is limited; 3 – any item can generate requirements for both the delivery and export of cargo; 4 – in the presence of several applications for delivery to one client, they are performed on different trips of the vehicle: 5 – the point of departure of the car may differ from the point of its return; 6 – the speed of the car is not the same on the arcs of the street-road network and can vary by the hour of the day; 7 - in the nodes of the road network there are delays of vehicles and they are not the same for different directions of movement (maneuvers). The criterion of optimality is the minimum of the total turnaround time on all routes, with mandatory fulfillment of all orders for transportation. In connection with the presence of the dynamic component in the conditions of the problem (the speed of the vehicle on the arcs of the street-road network), when the algorithms based on the Clark-Wright method are static, it is necessary to develop a modification of this method that takes into account the specifics of the problem formulation and to evaluate its effectiveness with respect to the original Clark-Wright method.

### **Description of the algorithm**

The algorithm proposed in this paper is based on the Clark–Wright method and assumes a modification of the formula for calculating the gains obtained as a result of routes fusion. The calculation formula of the gain from the inclusion of the i-j connection into the general system of routes ( $S_{ij}$ ) has the following form:

$$S_{ij} = t_{i - (n_i^k + 1)} + t_{0j} + t_{(z_j^m) - 0} - t_{ij} - t_{(z_j^m) - (n_i^k + 1)}, (1)$$

where  $t_{pq}$  is the travel time between the points with indices p and q; k is the index of the route to which the item with index i is included; m is the index of route to which the item with the index j ( $k \neq m$ ) is included;  $n_i^k$  is the ordinal

number of the point with the index i in the chain of the points of arrival of the route with the index k;  $z_j^m$  is the index of the last point in the chain of points of arrival of the route with the index m.

The variant of combining routes is selected by the maximum value of the gain  $S_{ij} \rightarrow \max$ . With a sequential implementation of the algorithm, the search for the maximum payoff is performed on the payout matrix line j, which implies the extension of the current route. With parallel implementation, the search for the maximum gain is carried out over the entire gain matrix. The calculation is over when all the gains are negative. The carrying capacity of the vehicle and the time of servicing the delivery points on the route (the time from the moment of arrival at the first point of delivery on the route until the moment of departure from the last point of delivery on the route) serve as limitations. The turnover time, being the most actual criterion for the optimality of the system of transport routes, is not the only parameter that evaluates the quality of solutions of the problem of transportation in practice. In addition to this indicator, when making a decision on transportation, the following additional parameters of the routes: mileage, km; turnover, tkm; the number of routes (vehicles).

When the time largely determines the speed of delivery of goods and the quality of service, the remaining indicators largely determine the cost of delivery of goods. Therefore, it was the combination of these indicators that made a comparative assessment of the quality of solutions of the test problem when using various methods.

## The efficiency of the algorithm

To evaluate the effectiveness of the proposed modification of the method, there was made a comparison of parameters of the route system designed using the Clark–Wright method in parallel (CWP) and sequential (CWC) implementations and a modified method, also in the parallel (MNP) and sequential (MNC) implementations. In some variants of the solution, the route systems were modified by applying the methods of local optimization of Lin (LL) [8] and Wren and Holiday (LWH) [9].

An array of seventy recipients, randomly located on an accurate street-road network model in the central part of Kharkiv, represented as an oriented graph was selected as a test problem. This model takes into account the changes in traffic speed on sections of the road network by the hour of the day and delays in the passage of intersections, depending on the type of maneuver (straight movement, turn to the right / left). The accuracy of the model is provided by the use of GPS markers in determining the coordinates of the vertices of the graph. The rates of movement by the hour of the day and the delay of vehicles during the passage of intersections are set at the levels determined as a result of a sample survey of the parameters of traffic flows. The speed of traffic (on the whole along the street-road network) varies within the limits of 10-60 km/h; the delays in the passage of intersections vary within the limits of 5–30 seconds. The volumes of delivery to recipients are set randomly in the range from 60 kg to 840 kg. When designing the routes, options for the use of vehicles of various carrying capacities, from 1,5 tons to 20,0 tons, were considered. The results of the solution of the test problem are given in Table 1. In this table, the numerator contains the absolute values of indicators, in the denominator - the deviations of the values of indicators from the base variant, which is the variant obtained using the Clark-Wright method in the parallel implementation (CWP).

As it can be seen from the given data, none of the compared algorithms provides stably better solutions, either for basic or additional indicators. Even the application of the sequential implementation of the compared algorithms with low vehicle carrying capacity provides a better solution than a parallel implementation of the Clark-Wright method, losing considerably to it at high load capacities of the vehicle. Concerning the number of routes, it should be noted that parallel implementations of algorithms ensure the design of a smaller number of routes than parallel implementations, with the carrying capacity of vehicles equal to 1,5 tons, 6,0 tons and 10,0 tons. With regard to such indicator as transport work, it should be noted that its importance in deciding on the route option can be decisive only if the parameters «turn-around time» and «mileage of the car» are equal. . The peculiarity of this indicator is that, the bigger number of arrival points is included in the route, the higher numerical value it acquires. If transport work is considered to be the only or basic indicator, then the optimal route system will consist only of pendulum routes.

Table 1	Indicators	of the	designed	routes	for the	test task
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Method	Route system parameters														
of solving the	Total turnaround time, h						Total mileage, km				Transport work, tkm				
test task	, ,						Vehicle capacity, tons								
	1,5	2,5	6,0	10,0	20,0	1,5	2,5	6,0	10,0	20,0	1,5	2,5	6,0	10,0	20,0
CWP	47,15	40,45	35,75	34,50	33,70	229,1	164,1	116,5	100,9	93,9	159,3	197,6	326,1	454,1	700,9
	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
CWP+LL	47,13	40,43	35,65	34,43	33,57	228,7	163,7	114,9	99,0	92,4	158,1	199,2	319,1	443,1	689,0
	-0,04	-0,04	-0,28	-0,19	-0,40	-0,18	-0,24	-1,38	-1,90	-1,56	-0,80	0,83	-2,15	-2,42	-1,70
CWP+LWH	47,15	40,42	35,62	34,47	33,55	228,7	164,2	115,2	99,3	92,4	157,7	195,3	317,2	443,0	689,9
	0,00	-0,08	-0,37	-0,10	-0,45	-0,16	0,05	-1,15	-1,65	-1,50	-1,03	-1,17	-2,73	-2,44	-1,57
CWP+LL+LWH	47,15	40,43	35,62	34,45	33,50	228,7	164,1	114,6	98,8	92,1	157,7	196,6	314,9	440,3	685,9
CWP+LL+LWH	0,00	-0,04	-0,37	-0,14	-0,59	-0,16	-0,02	-1,61	-2,11	-1,82	-1,03	-0,51	-3,44	-3,04	-2,14
CWC	47,05	40,65	36,13	34,85	33,45	240,2	,	128,8	109,8	97,1	184,4	224,0	367,9	572,5	829,3
	-0,21	0,49	1,07	1,01	-0,74	4,86	3,97	10,54	8,77	3,45	15,70	13,37	12,82	26,07	18,32
CWC+LL	47,03	40,58	36,08	34,75	33,47	240,0	170,3	127,4		96,7	184,6	215,3	362,8	570,1	823,5
CWC+LL	-0,25	0,33	0,93	0,72	-0,69	4,75	3,77	9,36	7,57	3,07	15,85	8,96	11,25	25,55	17,50
CWC+LWH	47,05	40,60	35,92	34,77	33,43	239,8	170,3	128,8	108,8	96,0	184,1	215,9	346,3	571,9	823,2
CWC+LWII	-0,21	0,37	0,47	0,77	-0,79	4,68	3,77	10,59	7,80	2,30	15,54	9,27	6,19	25,94	17,45
CWC+LL+LWH		40,60	36,08	34,73	33,45	239,8	170,3	127,4	107,4	95,6	184,0	215,9	361,7	563,6	814,6
	-0,21	0,37	0,93	,	-0,74	4,69	3,77	9,34	6,38	1,84	15,50	9,27	10,93	24,12	16,22
MNP	47,15	40,45	35,77	34,48	33,52	229,1	164,1	116,9	102,3	96,8	159,3	197,6	330,3	489,7	775,0
MINP	0,00	0,00	0,05	-0,05	-0,54	0,00	0,00	0,32	1,37	3,15	0,00	0,00	1,29	7,85	10,58
MNP+LL	47,13	40,43	35,58	34,33	33,27	228,7	163,7	114,2	98,8	93,7	158,1	199,2	321,0	454,3	754,4
	-0,04	-0,04	-0,47	-0,48	-1,29	-0,18	-0,24	-2,00	-2,10	-0,18	-0,80	0,83	-1,57	0,04	7,64
MNP+LWH	47,15	40,42	35,53	34,50	33,38	228,7	164,2	115,3	102,3	94,4	157,7	195,3	313,6	480,2	751,1
	0,00	-0,08	-0,61	0,00	-0,94	-0,16	0,05	-1,04	1,37	0,60	-1,03	-1,17	-3,84	5,74	7,17
MNP+LL+LWH	47,15	40,43	35,48	,	,	228,7	,	113,8	97,1	91,7	157,7	196,6	307,2	451,9	748,6
WINT TLLTLWII	0,00	-0,04	-0,75	-0,68	-1,38	-0,16	-0,02	-2,33	-3,78	-2,27	-1,03	-0,51	-5,79	-0,49	6,80
MNC	47,05	40,68	36,02	34,77	33,70	240,2	171,0	124,9	109,2	102,6	184,3	224,2	386,6	553,1	843,8
	-0,21	0,58	0,75	0,77	0,00	4,87	4,17	7,24	8,15	9,30	15,66	13,50	18,55	21,80	20,39
MNC+LL	47,03	40,58	35,97	34,63	33,57	240,0	170,3	124,1	106,6	99,0	184,6	215,3	385,1	538,5	809,2
	-0,25	0,33	0,61	0,39	-0,40	4,75	3,77	6,50	5,57	5,48	15,85	8,96	18,10	18,58	15,45
MNC+LWH	47,05	40,60	35,82	34,80	33,70	239,8	170,3	125,1	109,3	101,6	184,0	215,9	372,0	559,8	842,1
	-0,21	0,37	0,19	0,87	0,00	4,69	3,77	7,35	8,25	8,25	15,50	9,27	14,07	23,28	20,15
MNC+LL+LWH	47,05	40,60	35,95	34,70	33,53	239,8	170,3	124,1	106,5	98,4	184,0	215,9	385,4	538,4	813,1
	-0,21	0,37	0,56	0,58	-0,49	4,69	3,77	6,54	5,47	4,82	15,50	9,27	18,19	18,57	16,01

It is enough to compare the data of Table 1 to see that for the solution of the test task CWP, for automobiles with a carrying capacity of 1,5 tons and 20,0 tons the turnaround time is 47,15 h and 33,7 h, the milage 229,1 km and 93,9 km, and the transport work – 159,3 tkm and 700,9 tkm. That is, the obvious, by a several-fold factor, the advantage of vehicles with a carrying capacity of 20,0 tons by main indicators is accompanied by a large, also by a several-fold factor, increase in the volume of transportation. Taking into account the last remark, it can be noted that the modification of the Clark-Wright method proposed in the work provides a more qualitative solution of the test problem. The most effective in most cases is the parallel implementation of the method. In the same way, in most cases the application of local optimization methods allows improving the solutions obtained with the help of the basic algorithm. The deterioration of the solutions observed in some cases when using the local Wren–Holiday optimization method is explained by the geometric nature of this method. Here, an effective route is supposed to be the one whose route does not cross itself. However, in practice such a line of the route can lead to an increase in the number of left turns, which in turn leads to an increase in delays in the passage of intersections and, as a result, an increase in the turnover time.

#### Conclusions

Analysis of the experimental data makes it possible to point out the necessity of using the proposed modification of the Clark—Wright method. In the absence of clear advantages of any of the ways of designing routes, the software for solving this problem should provide the possibility of applying any variant of the algorithm imple-

mentation and using any of the local optimization procedures and the «manual» correction of the route system by the dispatcher.

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