ПРОБЛЕМЫ АВТОМОБИЛЬНО-ДОРОЖНОГО КОМПЛЕКСА

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VISCOELASTIC STRUCTURAL MODEL OF ASPHALT CONCRETE

V. Bogomolov, Prof., D. Sc. (Eng.), V. Zhdanyuk, Prof., D. Sc. (Eng.), A. Tsynka, P. G.,
Kharkov National Automobile and Highway University

Abstract. *The viscoelastic rheological model of asphalt concrete based on the generalized Kelvin model is offered. The mathematical model of asphalt concrete viscoelastic behavior that can be used for calculation of asphalt concrete upper layers of non-rigid pavements for strength and rutting has been developed. It has been proved that the structural model of Burgers does not fully meet all the requirements of the asphalt-concrete.*

Key words: *viscoelasticity, Kelvin model, Hooke element, Newton element, stress tensor, strain tensor.*

ВЯЗКОУПРУГАЯ СТРУКТУРНАЯ МОДЕЛЬ АСФАЛЬТОБЕТОНА

В.А. Богомолов, проф., д.т.н., В.К. Жданюк, проф., д.т.н., А.А. Цинка, асп.,
Харьковский автомобильно-дорожный университет

Аннотация. *Предложена вязкоупругая структурная модель асфальтобетона, построенная на обобщенной модели Кельвина. Разработана математическая модель такой схемы. Она может использоваться при расчетах верхних несущих слоев нежестких дорожных одежд (асфальтобетона) на прочность и колееобразование.*

Ключевые слова: *вязкоупругость, модель Кельвина, элемент Гука, элемент Ньютона, тензор напряжений, тензор деформаций.*

В'ЯЗКОПРУЖНА СТРУКТУРНА МОДЕЛЬ АСФАЛЬТОБЕТОНУ

В.О. Богомолов, проф., д.т.н., В.К. Жданюк, проф., д.т.н., А.О. Цинка, асп.,
Харківський національний автомобільно-дорожній університет

Анотація. *Запропонована в'язкопружна структурна модель асфальтобетону, в основу якої покладена узагальнена модель Кельвіна. Розроблено математичну модель такої схеми. Вона може використовуватись для розрахунків верхніх несучих шарів нежорсткого дорожнього одягу (асфальтобетону) на міцність та колієутворення.*

Ключові слова: *в'язкопружність, модель Кельвіна, елемент Гука, елемент Ньютона, тензор напружень, тензор деформацій.*

Introduction

Currently, in Ukraine according to the regulatory document [1] the concept of the so-called elastic half-space [2, 3] based on the classical theory of elasticity [2] is used in calculation of non rigid pavements. However, with such an approach the researchers of non-rigid pavements revealed a number of experimentally established facts that clearly contradict the theory of elasticity.

Only some of them are as follows:
– many types of asphalt concrete in pavement layers are prone to rutting [24];
– the module of asphalt concrete elasticity depends on the rate of deformation [1, 6].

These and other signs point to the fact that asphalt concrete is a viscoelastic [7–13], not purely elastic material [3].

Due to this the task of developing a reliable and practically convenient model of asphalt concrete viscoelastic behavior appears urgent [14].

Analysis of publications

Various researchers considered a number of structural rheological models to describe the viscoelastic behavior of bitumen and asphalt concrete. Some of them are presented in fig. 1.

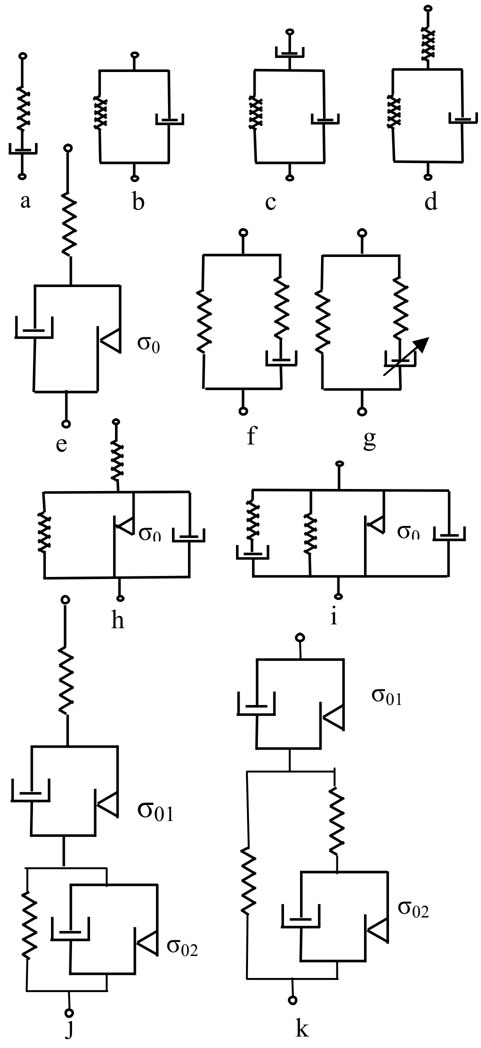

Fig. 1. Structural rheological models: a – Maxwell model [12, 15]; b – Kelvin-Voigt model [12, 15, 22]; c – Prandtl model [11]; d – standard body [17, 21]; e – Shvedov-Bingham model [16]; f – Poynting-Thomson model [18]; g – V.O. Zolotaryev model [17]; h – Ya.M. Yakimenko model [18]; i – A.M. Boguslavskiy model [7]; j – k – KhNAHU models [14, 19]

The basic requirements to the asphalt concrete rheological model have been put forward in [13]. Simulation of asphalt concrete work with its use should provide for determination of creeping, relaxation, retardation, residual strains etc. However, many authors believe that for meeting these requirements it is quite sufficient to accept the so-called Burgers model [7, 11, 12, 17, 22], shown in fig. 2.

Goal and task-setting

The goal of the given work consists in building a rheological model of asphalt concrete suitable when used for analyzing the strain-stress state of non-rigid road pavement layers.

Burgers structural rheological model

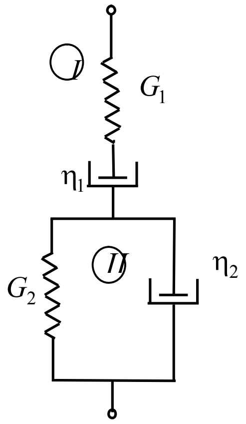

Fig. 2. Burgers structural rheological model: G_1, G_2 – stiffness of Hooke elements; η_1, η_2 – viscosity of Newton elements; I – Maxwell link; II – Kelvin link

In KhNAHU the methods for calculation [14] of parameters of the rheological model shown in Fig. 2, based on experimentally established correspondences for asphalt concrete creeping have been developed (see. fig. 3).

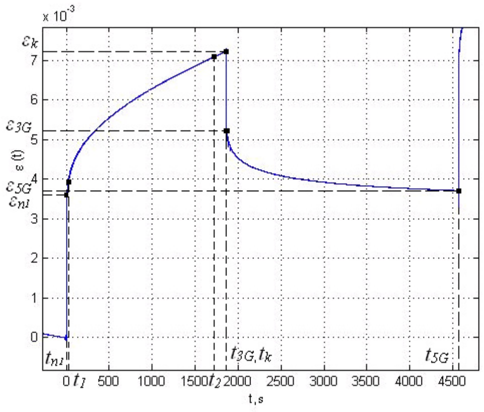

Fig. 3. Deformation of the asphalt concrete sample during tests for creeping

Experimental studies were conducted on a specially designed stand, whose scheme is given in fig. 4, and the power unit of the laboratory equipment – in fig. 5.

Experimentally obtained values of viscoelastic properties of the rheological model according to fig. 2 for macadam-mastic asphalt concrete of SHCHMA 15 type based on bitumen of BND 90–130 grade at 20°C are given in table 1.

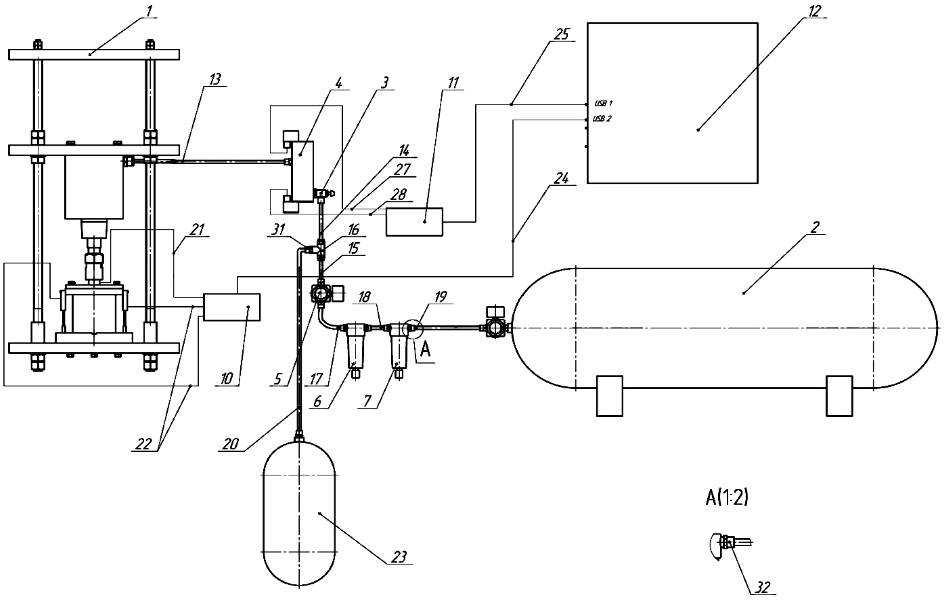

Fig. 4. Scheme of the stand for determining asphalt concrete viscoelastic characteristics under compression: 1 – device for testing asphalt concrete creeping under compression; 2 – compressor; 3 – pneumatic throttle; 4 – pneumatic distributer with electromagnetic coils; 5 – pressure regulator; 6, 7 – filters; 10 – analog-digital converter of a signal from sensors; 11 – unit of control for pneumatic distributer coils; 12 – unit for processing and recording signals from sensors; 13–15, 17–20 – pneumatic wires; 16 – T-joint; 21, 22, 24, 25, 27, 28 – electric wires; 23 – receiver; 31, 32 – fittings

Table 1 Values of variables G_1, G_2, η_1, η_2 for asphalt concrete of SHCHMA 15 type based on bitumen of BND 90-130 grade (time of sample loading is 0.1s, time of keeping under load see fig. 3)

Value of compression strain in asphalt concrete under uniaxial loading	η_1, Pa·s	G_1, Pa	η_2, Pa·s	G_2, Pa	$[\varepsilon_k - \varepsilon_{3G}]^*$	$[\varepsilon_{(2)k} - \varepsilon_{(2)3G}]^{**}$
0,8 MPa	$1.48 \cdot 10^{11}$	$1.67 \cdot 10^{8}$	$2.00 \cdot 10^{9}$	$1.10 \cdot 10^{8}$	$2.19 \cdot 10^{-3}$	$3.57 \cdot 10^{-5}$
0,6 MPa	$6.92 \cdot 10^{11}$	$1.69 \cdot 10^{8}$	$2.24 \cdot 10^{8}$	$1.29 \cdot 10^{8}$	$1.70 \cdot 10^{-3}$	$1.05 \cdot 10^{-4}$

Notes. * – strains marked in Fig. 3; ** – Calculated strains correspond only to strains in Kelvin element (η_2, G_2) with applying compression force for 0.1 sec.

According to the results of tests the following conclusions can be made:

– under compression strains with the value less than 0.6 MPa, the deformation pattern of the asphalt concrete under study does not correspond to the calculated model in fig. 2. This is proved by the fact that there is virtually no plot $\varepsilon_k - \varepsilon_{3G}$ on oscillograms (see fig. 3), which corresponds to an elastic stage of deformation. It is almost impossible to calculate value G_1 (see fig. 2);

– values G_1 in Table 1 are of such a magnitude that it is impossible to receive values of elasticity module normalized in [1] (for the tested material it reaches the value of $1.2 \cdot 10^9$ MPa) when compression force is applied for 0.1 s. This is evident from the difference of values $[\varepsilon_k - \varepsilon_{3G}]$ >> $[\varepsilon_{(2)k} - \varepsilon_{(2)3G}]$.

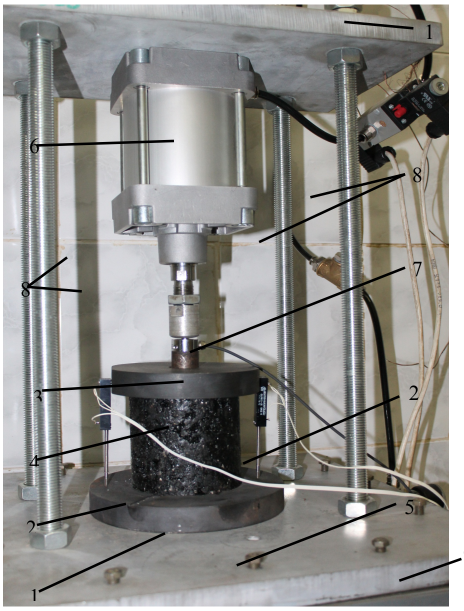

Fig. 5 – Power unit of the laboratory equipment for testing asphalt concrete: 1 – asphalt concrete sample; 2 – sensors of motion; 3 – sensor of force; 4 – pressure foot; 5 – bearing foot; 6 – pneumatic cylinder; 7 – tip; 8 – supporting bars; 9 – lower base plate; 10 – upper base plate

It means that the mentioned module, depending on the time of load application (in this case not less than 0.1 s) cannot vary by more than 10%. At the same time according to [1] the adjusting factor is calculated by formula

$$K_t = \sqrt[3]{\frac{t_{\text{э}}}{t_{0.1}}}\,, \qquad (1)$$

where $t_{0.1} = 0.1$ s; $t_{\text{э}}$ – the real time of sample loading.

For example, if we assume that $t_{\text{э}} = 0.6$ s, then $K_t = 1.82$. That is, mismatch of the model in fig. 2 to the requirements [1] is obvious;

– conditionally during deformation in fig. 3 we distinguish two phases:

the 1st one takes place when the time of loading increases, i.e. when

$$t = (0\ldots0.1)\ \text{s}, \qquad (2)$$

we call this period the instant strains; the 2nd phase takes place during the whole process of loading, when

$$t = (0\ldots1800\text{–}2000)\ \text{s}. \qquad (3)$$

We call it a period of long-term creeping.

Studies have shown that for the best approximation of the process of deformation within the range (2), in particular, to almost meet the condition (1) it is necessary to select the Kelvin link in fig. 2 with the time of delay [16]

$$\tau = \frac{\eta_2}{G_2} \cong (0.1\ldots0.2)\ \text{s}. \qquad (4)$$

For the best approximation of the deformation process within the range (3), it is necessary that

$$\tau > (10\ldots15)\ \text{s}. \qquad (5)$$

Five-element rheological structural model

The significant difference between ranges (4) and (5) does not allow to choose τ such that could simultaneously meet the conditions of instant strain and long-term creeping.

The above shortcomings can be removed if in further research instead of the model in fig. 2 the five-element structural model shown in fig. 6 will be accepted.

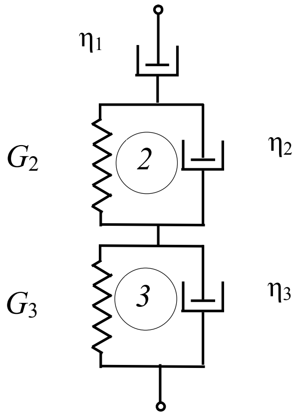

Fig. 6. Five-element structural viscoelastic model of asphalt concrete: G_2, G_3 – stiffness of relevant Hooke elements: η_1, η_2, η_3 – viscosity of relevant Newton elements; 2, 3 – numbers of Kelvin elements

Let us specify some features of the model shown in fig. 6:

– one should not confuse viscosity and stiffness in the models in fig. 2 and fig. 6. Further in the text all indexes with G and η correspond to fig. 6;

– based on (4), (5) there is a following ratio in fig. 6:

$$\frac{\eta_2}{G_2} << \frac{\eta_3}{G_3} \qquad (6)$$

– the model in fig. 6 is a particular case of the generalized Kelvin model [23];

– the model in fig. 6 proceeds from fig. 2 by means of parallel joining the «weak» Newton element to the free Hooke element (fig. 2). This ensures proper work of the physical model in the range (2);

– with η_2=0 the model in fig. 6 is degenerated into the Burgers model in fig. 2. Thus, the model in fig. 2 is a particular case of the model in fig. 6;

– the particular case of the generalized Maxwell model [23] shown in fig. 7 [24] can be the analogue of the model in fig. 6.

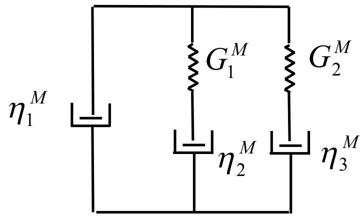

Fig. 7. Analog of the structural rheological model in fig. 6: G_1^M , G_2^M – stiffness of relevant Hooke elements; η_1^M, η_2^M, η_3^M – viscosity of relevant Newton elements

For asphalt concrete under study with the use of methods developed in KhNAHU the values required for the structural model in fig. 6 that are given in table 2 were defined and calculated.

Table 2 Values of magnitudes , G_2 , G_3 , η_1 , η_2 , η_3 at 20°C for asphalt concrete of the SHCHMA 15 type

Value of compression strain in asphalt concrete under uniaxial loading	η_1 , Pa·s	G_2 , Pa	η_2 , Pa·s	G_3 , Pa	η_3 Pa·s	$E^*_{0,6}$ Pa	$E^*_{0,1}$ Pa
0,8 MPa	$1{,}58 \cdot 10^{11}$	$1{,}18 \cdot 10^{8}$	$1{,}59 \cdot 10^{7}$	$3{,}03 \cdot 10^{8}$	$4{,}49 \cdot 10^{9}$	$3{,}39 \cdot 10^{8}$	$9{,}24 \cdot 10^{8}$
0,6 MPa	$6{,}7 \cdot 10^{11}$	$1{,}15 \cdot 10^{8}$	$1{,}05 \cdot 10^{7}$	$3{,}42 \cdot 10^{8}$	$3{,}13 \cdot 10^{9}$	$2{,}91 \cdot 10^{8}$	$6{,}86 \cdot 10^{8}$
0,4 MPa	$9{,}48 \cdot 10^{11}$	$2{,}2 \cdot 10^{8}$	$1{,}51 \cdot 10^{7}$	$2{,}85 \cdot 10^{8}$	$4{,}47 \cdot 10^{9}$	$5{,}11 \cdot 10^{8}$	$1{,}1 \cdot 10^{9}$
0,2 MPa	$1{,}35 \cdot 10^{12}$	$7{,}41 \cdot 10^{7}$	$1{,}86 \cdot 10^{8}$	$3{,}3 \cdot 10^{8}$	$4{,}47 \cdot 10^{9}$	$1{,}48 \cdot 10^{9}$	$8{,}13 \cdot 10^{9}$

Note. * – Calculated modules of elasticity with the duration of loading growth for 0.6 s and 0.1 s respectively

Mathematical model of the five-elemental scheme

It is more convenient to write the mathematical model from the scheme in fig. 6 for the strain tensor [25]

$$T_d = D_d + I\varepsilon_{av}, \qquad (7)$$

where D_d – a strain deviator; I – an identity matrix;

$$\varepsilon_{av} = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3}, \qquad (8)$$

where: ε_x, ε_y, ε_z – relative strains by axes of the accepted Cartesian reference system.

For the Kelvin link the differential equations [25] are known

$$D_H = 2G_n D_d^{(n)} + 2\eta_n \cdot \dot{D}_d^{(n)}, \ n = 2,\ 3, \qquad (9)$$

where: D_H – the stress deviator of the Kelvin link; $D_d^{(n)}$, $\dot{D}_d^{(n)}$ is the strain deviator and its first derivative by time for Kelvin n -th link (see fig. 6); G_n, η_n - are stiffness and viscosity of Hooke and Newton elements in Kelvin n -th link;

$$\frac{G_n}{\eta_n}\varepsilon_{ar}^{(n)} + \dot{\varepsilon}_{av}^{(n)} = \frac{1-2\mu_n}{2(1+\mu_n)\eta_n}\sigma_{av}, \qquad (10)$$

where $\varepsilon_{av}^{(n)}$, $\dot{\varepsilon}_{av}^{(n)}$ – an average strain and it's first derivative by time for Kelvin n -th link; μ_n – the coefficient of transverse strain for Kelvin n -th link; σ_{av} – average stress [25].

For Newton element it is known that [25]

$$D_H = 2\eta_1 \dot{D}_d^{(1)}; \qquad (11)$$

$$\overset{\bullet}{\varepsilon}{}_{av}^{(1)} = \frac{1-2\mu_1}{2(1+\mu_1)\eta_1}\sigma_{av}, \qquad (12)$$

where index 1 means belonging of the parameter to the Newton element in fig. 6; μ_1 – is the coefficient of transverse strain of this Newton element.

From (9) – (12) the following can be obtained

$$D_d = \sum_{n=2}^{3}[D_{d0}^{(n)} \cdot e^{-\frac{t}{\tau_n}} + \frac{1}{2\eta_n}\int_0^t D_H(\xi) \cdot e^{-\frac{t-\xi}{\tau_n}} d\xi] +$$

$$+ D_{d0}^{(1)} + \frac{1}{2\eta_1}\int_0^t D_H(\xi) d\xi; \qquad (13)$$

$$\varepsilon_{av} = \sum_{n=2}^{3}[\varepsilon_{av0}^{(n)} \cdot e^{-\frac{t}{\tau_n}} + \frac{1-2\mu_n}{2(1+\mu_n)\eta_n}\int_0^t \sigma_{cp}(\xi) \cdot e^{-\frac{t-\xi}{\tau_n}} d\xi] +$$

$$+ \varepsilon_{av0}^1 + \frac{1-2\mu}{2(1+\mu_1)\eta_1}\int_0^t \sigma_{av}(\xi) d\xi, \qquad (14)$$

where D_{d0}, $\varepsilon_{av0}^{(n)}$, $n = 2,\ 3$ – the initial parameters of the process of strain; t – the time of the process of deformation; ξ – the current time of the process;

$$\tau_n = \frac{\eta_n}{G_n}, \ n = 2,\ 3. \qquad (15)$$

The typical graph of the approximation of the experimental data is presented in fig. 8.

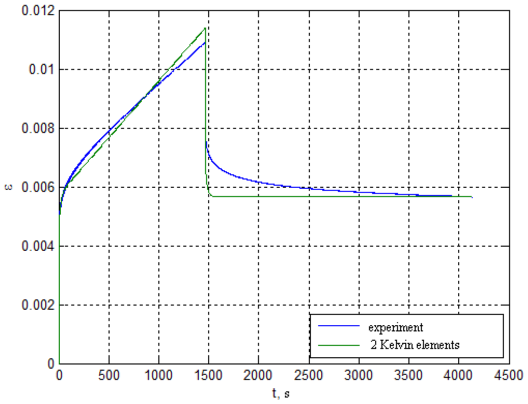

Fig. 8. The typical graph of approximation of experimental data

Conclusions

1. It is most appropriate to present the structural viscoelastic model of asphalt concrete in the form of two Kelvin links connected in series and one Newton element.

2. One of the Kelvin links should ensure satisfactory operation of the model in the mode of instant strains at $\tau \leq 0.2$ s while the other – in the mode of long-term creeping at $\tau \geq 10$ s.

The proposed mathematical model allows:
– to increase the number of Kelvin links, i.e. in (13), (14) there may be $n > 3$, that will improve the degree of approximation of the experimental curve of creeping;
– if necessary to include the free Hooke element connected in series into the model in fig. 6. if its stiffness is taken equal to G_1, then in expression (13) with the sign «+» the component is added

$$\frac{D_H}{2G_1}, \qquad (16)$$

in expression (14) also with the sign «+» the component is added

$$\frac{1-2\nu}{2(1+\nu)G_1}\sigma_{av}, \qquad (17)$$

where ν – Poisson's ratio of the Hooke free e lement.

This can become necessary while studying asphalt concrete at e.g. low temperatures.

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Рецензент: В.П. Кожушко, профессор, д.т.н., ХНАДУ.

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