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Maximum automobile acceleration

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Abstract. Problem. *The disadvantage of current dependences for determining the acceleration indicators at engine maximum brake power and driving tire-to-surface friction coefficients is that they are adequate only if the engine and transmission parameters provide power input to the drive wheels rolling without slipping regardless to speed. To eliminate this drawback, it is necessary to take into account that the power input to the drive wheels depends on the engine shaft speed, and therefore on the speed of the vehicle when accelerating.* ***Goal.*** *The purpose of the work is to further develop the theory of the automobile by improving the dependencies that allow determining the automobile acceleration rates and assessing the nature of its acceleration process from the design factors.* ***Methodology.*** *The approaches taken to achieve this goal are based on laws of physics, theoretical mechanics and the theory of automobile.* ***Results.*** *Analytic dependences for determining maximum and limiting automobile acceleration when speeding up depending on its design factors and speed have been improved. Dependences for determining the range of drive wheel slipping on the automobile speed when accelerating and the limiting automobile acceleration under the condition of its pitch stability have been obtained. When studying the automobile acceleration process theoretically it was found that the developed dependences allow determining the nature of automobile movement and assessing the influence of its design factors on the acceleration indicators.* ***Originality.*** *The obtained dependences for determining the maximum and limiting acceleration, the range of driving speeds with wheel slip when automobile accelerating allowed us to clarify the idea of the nature of movement during acceleration and the influence of automobile design factors on the acceleration indicators.* ***Practical value.*** *The obtained dependences can be used in designing new and improving racing cars such as dragsters, and analysing the dynamics of the vehicle when accelerating with full fuel delivery and determining the nature of driving tire-to-surface friction depending on the driving speed.*

Key words: *record racing car, acceleration, limiting acceleration, friction coefficient, gravity centre coordinates, horsepower, acceleration characteristic.*

Introduction

When the vehicle is moving, its maximum acceleration is limited by many factors. For record racing cars with linear motion on specially prepared sections, the maximum acceleration is limited only by the maximum engine power, maximum driving tire-to-surface friction and resistance to the rollover in the longitudinal plane. The engine maximum brake power of a record racing car for a given friction coefficient usually provides the possibility to approach and overcome the limits of maximum driving tire-to-surface friction. In this case, the maximum acceleration $j_{\max}$ reaches the value of limiting acceleration $j_{\lim}$. The motion of the car with maxi mum acceleration $j_{\lim}$ is possible only up to the speed $v_{j\lim}$ at which tractive effort on driving wheels becomes equal to the tire-to-surface friction. The values of limiting acceleration $j_{\lim}$ and speed $v_{j\lim}$ for a given friction coefficient φ_x depend on the gravity centre coordinates.

Literature review for determining maximum and limiting automobile acceleration

The maximum and limiting automobile acceleration is usually calculated using the formulas [1, 2, 4]:

$$j_{\max} = \frac{D_{\max} - f}{\delta_{\text{вр}}} \cdot g; \; j_{\lim} = D_{\varphi\max} \cdot g. \quad (1)$$

where $D_{\max}$, $D_{\varphi\max}$ are dynamic factors for traction and friction respectively; f is the rolling resistance coefficient; $\delta_{\text{вр}}$ is automobile rotational inertia coefficient; g is the gravitational acceleration.

In [5], the author uses a different form of writing formulas (1), but their essence is the same.

The authors of the works [2–4, 8–10] determine the maximum dynamic factor in terms of driving wheel traction and friction using the dependences:

$$D_{\max} = \frac{P_{\text{к}\max} - P_{\text{в}}}{G_{\text{a}}}; \; D_{\varphi\max} = \frac{P_{\varphi} - P_{\text{в}}}{G_{\text{a}}}, \quad (2)$$

where $P_{\text{кmax}}$, P_{φ}, $P_{\text{в}}$, G_{a} are the maximum traction force on driving wheels, driving tire-to-surface friction, air resistance and gravity.

Usually, the value of air resistance $P_{\text{в}}$ in formula (2) and the rolling resistance coefficient f in (1) are ignored because at $D_{\max}$ ($D_{\varphi\max}$) the driving speed is low and the values of these parameters are small [1, 4, 8].

The driving tire-to-surface friction P_{φ} is calculated using the formulas [1, 3, 4, 9]:

$$P_{\varphi} = R_z \cdot \varphi_x \text{ or } P_{\varphi} = m_2 \cdot G_{\text{в}} \cdot \varphi_x, \quad (3)$$

where R_z is the normal dynamic response of the road on the wheels of the driving axle; m_2 is coefficient of changing normal response on the rear axle; $G_{\text{в}}$ is static load on the driving axle.

The author [5] obtained a dependence for determining the maximum driving tire-to-surface friction where changes in the normal load are taken into account by the coefficients of changing the normal reaction for the rear driving and front driven axles:

$$m_2 = \frac{1}{1 - \varphi \cdot \frac{r}{L}}; \quad m_1 = \frac{1 - \varphi \cdot \frac{r}{b}}{1 - \varphi \cdot \frac{r}{L}}, \quad (4)$$

where r is the radius of the wheel (in [5], it is assumed that r is equal to the height of the gravity centre h_g); φ is the tire-to-road friction coefficient; L is vehicle base; b is rear longitudinal coordinate of the automobile gravity centre.

The author of the study [5] obtained a dependence on the automobile power balance for determining the automobile driving speed $v_{j\lim}$, to which the engine is capable of providing the limiting value of acceleration $j_{\lim}$, i.e., fulfilment of the condition $P_{\text{кmax}} \geq P_{\varphi}$:

$$v_{j\lim} = \frac{75 N_{e\max} \cdot \eta_{\text{тр}}}{\delta_{\text{tm}} \cdot P_{\text{к}\max}}, \quad (5)$$

where $N_{e\max}$ is the maximum engine power, cc; η_{tr} is transmission efficiency; δ_{tm} is rotational inertia coefficient.

Formula (5) was obtained under the assumption that the power consumed to overcome air and road resistance is insignificant and is ignored. In formula (5), the author assumes that $P_{\text{кmax}}=P_{\varphi}$ the study

The disadvantage of equation (5) is that it contains the rotational inertia coefficient, but under the condition $P_{\text{кmax}}=P_{\varphi}$, the rotational inertia becomes internal force and does not affect the automobile acceleration and driving speed. In addition, formula (5) is adequate only if the engine and transmission parameters ensure the power input to the drive wheels rolling without slipping regardless of the speed.

Therefore, the dependence for determining the driving speed $v_{j\lim}$ up to which the automobile can move with maximum acceleration, obtained by the author of work [5], requires correction.

Equation of automobile motion with full use of tire-to-surface friction properties

In the general case of automobile motion, the equation is of the form [1-4, 6]:

$$P_{\text{к}} - P_{\psi} - P_{\text{a}} - P_j = 0. \quad (6)$$

where $P_{\text{к}}$ is the total traction on the driving wheels, N; P_{ψ} is road resistance, N; P_{a} is air resistance, N; P_j is resistance to automobile acceleration.

The limiting automobile acceleration is limited by the friction condition, so the total traction $P_{\text{к}}$ is equal to the friction $P_{\square}$ on the driving wheels. As the limiting automobile acceleration is determined on a horizontal section, it can be assumed that road resistance is equal to the automobile rolling resistance, i.e. $P_{\psi} = P_f$. The equation of automobile motion with limiting acceleration is of the form:

$$P_{\varphi 2} - P_{f1} - P_a - P_j^{'} = 0, \quad (7)$$

where $P_{\varphi 2}$ is driving rear tire-to-surface traction, N; P_{f1} is resistance to the front driven wheel rolling, N; $P_j^{'}$ is automobile progressive rotational inertia.

The rear driving tire-to-surface friction is determined using the formula [1-4, 6, 7]:

$$P_{\varphi 2} = R_{z2} \cdot \varphi_x, \quad (8)$$

where R_{z2} is the normal response on the wheels of the rear driving axle when the vehicle is moving at a speed v; φ_x is the rear driving tire-to-surface friction coefficient.

The resistance to the front driven wheel rolling is determined by the formula [1-4, 6, 7]:

$$P_{f1} = R_{z1} \cdot f_1, \quad (9)$$

where R_{z1} is the normal response on the front (driven) axle wheels when the vehicle is moving at a speed v.

The air resistance is determined by the formula [1-4, 6]:

$$P_в = к_в \cdot F \cdot v^2, \quad (10)$$

where $к_в$ is the air resistance coefficient, $(N{\cdot}c^2)/m^4$; F is the frontage area of the vehicle, m^2; v is vehicle speed, m/s.

We substitute the values of the forces in (6) and the equation of automobile motion with limiting acceleration will be of the form:

$$R_{z2} \cdot \varphi_x - R_{z1} \cdot f_1 - \\ -к_в \cdot F \cdot v^2 - \frac{G_a}{g} \cdot j_{\lim} = 0. \quad (11)$$

To determine the limiting speed at a given friction coefficient φ_x, it is necessary to know the values of the normal responses on the axes R_{z1}, R_{z2} and the resistance coefficient of wheel rolling f_1 when moving at the limiting speed.

It is known that the resistance coefficient of wheel rolling f_1 depends on the tire parameters and the driving speed. There are many empirical dependences for determining the rolling resistance coefficient [6, 7, 11-13]. At speeds of more than 400 km/h, we suggest using the empirical dependence as a function of the vehicle speed and the internal tire pressure [5]:

$$f_1 = \frac{1}{p^{0.64}} \cdot \left(20 + \frac{v_a^{3.7}}{1294000 \cdot p^{1.44}}\right), \quad (12)$$

where f_1 is the rolling resistance in tires, kg. per ton of weight; p is the internal tire pressure, kg/cm^2; v_a is vehicle speed, km/h.

To use formula (12) for determining the rolling resistance coefficient it is necessary to transform it into the following one:

$$f_1 = \frac{1}{1000\,p^{0.64}} \cdot \left(20 + \frac{v_a^{3.7}}{1294000 \cdot p^{1.44}}\right). \quad (13)$$

where f_1 is the coefficient of resistance to the front axle wheel rolling.

If the car accelerates over a short distance, its speed is less than 400 km/h, therefore, when calculating, one can use a simplified formula to determine the rolling resistance coefficient [5]:

$$f = f_0 + 0.5 \cdot 10^{-6} \cdot v_a^2, \quad (14)$$

where f_0 is the rolling resistance coefficient at low speed.

Normal responses on the automobile axles with full use of the driving tire-to-surface friction

The value of maximum driving tire-to-surface friction depends on the traction coefficient and road normal responses. Therefore, determining normal responses on the drive wheels is extremely important to assess maximum automobile acceleration.

To determine the normal responses on the axles of the vehicle, consider the diagram shown in Figure 1. The automobile motion at the maximum speed is uniform motion. That's why inertia does not influence the vehicle. In this case, the motion occurs under the action of external forces: road responses R_{x1}, R_{x2} and R_{z1}, R_{z2}, air resistance $P_в$, automobile gravity G_a. For record racing cars, we can assume that the height $h_в$ is approximately equal to the height of the gravity centre h_g:

$$R_{z1} \cdot a - R_{x1} \cdot h_g + R_{x2} \cdot h_g - R_{z2} \cdot b = 0. \quad (15)$$

To determine normal responses reactions, we set up the equation of moments relative to the automobile gravity centre.

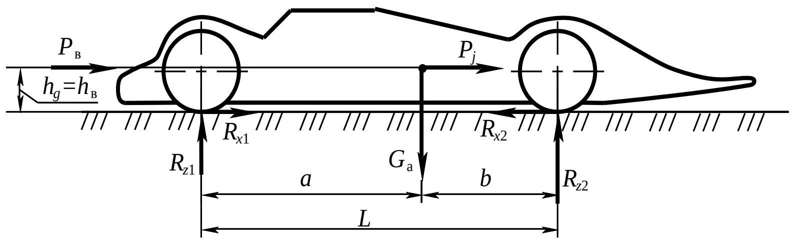

Fig. 1. The diagram of forces influencing on an automobile when accelerating: a, b, h_q are gravity centre coordinates; $h_в$ is the height of the centre of effort of the automobile; $G_а$ is the weight of the vehicle; R_{x1} is the linear response on the front wheels; R_{x2} is the linear response on the rear wheels, L is the vehicle base, m

In equation (15), we define R_{z1} by means of R_{z2}:

$$(G_а - R_{z2}) \cdot a - R_{x1} \cdot h_g + R_{x2} \cdot h_g - R_{z2} \cdot b = 0 \quad . \qquad (16)$$

After transformations, equation (16) will be of the form:

$$G_а \cdot a - R_{z2} \cdot L - R_{x1} \cdot h_g + R_{x2} \cdot h_g = 0 . \qquad (17)$$

From equation (17) we determine the normal response on the rear axle of the vehicle:

$$R_{z2} = G_а \cdot \frac{a}{L} - R_{x1} \cdot \frac{h_g}{L} + R_{x2} \cdot \frac{h_g}{L} . \qquad (18)$$

The limiting responses R_{x1} and R_{x2} for uniform motion are determined by the equations:

$$R_{x1} = R_{z1} \cdot f_1 , \qquad (19)$$

where f_1 is the coefficient of resistance to the front axle wheel rolling:

$$R_{x2} = P_{\varphi 2} = R_{z2} \cdot \varphi_x , \qquad (20)$$

where φ_x is the driving tire-to-surface friction coefficient.

Substitute (19) and (20) into equation (18):

$$R_{z2} = G_а \cdot \frac{a}{L} - R_{z1} \cdot f_1 \cdot \frac{h_g}{L} + R_{z2} \cdot \varphi_x \cdot \frac{h_g}{L} . \qquad (21)$$

Considering that $R_{z1} = G_а - R_{z2}$, equation (21) will be of the form:

$$R_{z2} = G_а \cdot \frac{a}{L} - (G_а - R_{z2}) \times \times f_1 \cdot \frac{h_g}{L} + R_{z2} \cdot \varphi_x \cdot \frac{h_g}{L} , \qquad (22)$$

after transformations of equation (22) we determine the normal response on the rear axle:

$$R_{z2} = G_а \cdot \frac{a - f_1 \cdot h_g}{L - (\varphi_x + f_1) \cdot h_g} . \qquad (23)$$

To determine the normal responses on the front axle in equation (15) we determine R_{z2} by means of R_{z1} and after similar transformations we get:

$$R_{z1} = G_а \cdot \frac{b - \varphi_x \cdot h_g}{L - (\varphi_x + f_1) \cdot h_g} . \qquad (24)$$

Maximum and limiting automobile acceleration

The diagram of the forces influencing the automobile when accelerating is shown in Fig. 1. To determine the limiting automobile acceleration, we use the equation of moments relative to the centre of the front tire-to-surface friction. In this case, we assume that $h_в = h_g$:

$$(P_в + P_j) \cdot h_g + G_а \cdot a - R_{z2} \cdot L = 0 , \qquad (25)$$

we transform (25) into the following form:

$$P_j \cdot h_g = R_{z2} \cdot L - G_а \cdot a - P_в \cdot h_g . \qquad (26)$$

Taking into account (23), equation (26) is of the form:

$$P_j \cdot h_g = G_а \cdot \frac{a - f_1 \cdot h_g}{L - (\varphi_x + f_1) \cdot h_g} \times$$
$$\times L - G_а \cdot a - P_в \cdot h_g. \qquad (27)$$

In the denominator of the first term on the right-hand side of (27), we put L outside the brackets and after transformations we obtain:

$$P_j \cdot h_g = G_а \cdot \frac{a - f_1 \cdot h_g}{1 - (\varphi_x + f_1) \cdot \frac{h_g}{L}} - G_а \cdot a - P_в \cdot h_g. \qquad (28)$$

In equation (28), we reduce the first two terms on the right-hand side to a common denominator and perform the transformation:

$$P_j \cdot h_g = G_а \cdot \frac{(\varphi_x + f_1) \cdot \frac{a}{L} - f_1}{1 - (\varphi_x + f_1) \cdot \frac{h_g}{L}} \cdot h_g - P_в \cdot h_g. \qquad (29)$$

It is worth explaining the forces P_j and $P_в$. It should be noted that resistance to automobile acceleration P_j with full use of the driving tire-to-surface friction is equal to the automobile progressive rotational inertia. In this case, the rotational inertia of the driving wheels and the rotating masses that are connected to them becomes internal, therefore it does not affect the automobile acceleration. The rotational inertia of the driven wheels is not significant; therefore, it can be ignored. Taking into account this assumption, after transformations, we obtain:

$$\frac{G_а}{g} \cdot j_{\lim} = G_а \cdot \frac{(\varphi_x + f_1) \cdot \frac{a}{L} - f_1}{1 - (\varphi_x + f_1) \cdot \frac{h_g}{L}} - к_в \cdot F \cdot v^2. \qquad (30)$$

From equation (30) we obtain the value of the limiting automobile acceleration depending on its design factors and motion parameters:

$$j_{\lim} = \left[\frac{(\varphi_x + f_1) \cdot \frac{a}{L} - f_1}{1 - (\varphi_x + f_1) \cdot \frac{h_g}{L}} - \frac{к_в \cdot F \cdot v^2}{G_а \cdot} \right] \cdot g. \qquad (31)$$

The first term in braces characterizes the dependence of limiting acceleration on the specific driving tire-to-surface friction, which is a function of the driving tire-to-surface friction coefficient, the rolling resistance coefficient of the front axle wheels and the gravity centre coordinates. The second term characterizes the specific air resistance, which is directly proportional to the streamlining factor of the vehicle and is opposite to its weight and is a quadratic function of the driving speed. That is, the influence of the streamlining factor on the automobile acceleration decreases with increasing its weight, and increases with increasing its speed. The limiting automobile acceleration $j_{\lim}$ can be increased by a rational choice of the gravity centre coordinates b, h_g and the base L to the absolute physical limit of acceleration $j_{\phi\lim}$ at a given friction coefficient φ_x. The physical limit of acceleration is determined by preserving the longitudinal stability of the vehicle. Disarrangement of the longitudinal automobile stability occurs due to the automobile rollover around the centre of the rear axle tire-to-surface friction. contact of the wheels of. To determine the ratio of the gravity centre coordinates b, h_g and the base L, at which it is possible to achieve the limiting acceleration on the edge of rollover $j_{[\lim]}$, consider the moment balance of overturning and stabilizing the positions of the car relative to the centre of rear tire-to-surface friction. In this case, according to Figure 1, the moment overturning the car is determined by the dependence:

$$M_{пер} = (P_j + P_в) \cdot h_g =$$
$$= \left(\frac{G_а}{g} \cdot j_{[\lim]} + к_в \cdot F \cdot v^2 \right) \cdot h_g, \qquad (32)$$

where $j_{[\lim]}$ is the limiting acceleration at the limit of automobile rollover.

The moment stabilizing the position of the vehicle in the vertical plane is formed by the gravity:

$$M_{ст} = G_а \cdot b. \qquad (33)$$

The second term in brackets of equation (32) is insignificant, as the limiting acceleration at the limit of automobile rollover $j_{[\lim]}$ is realized at low speed v, so it can be ignored. With the accepted assumption, the moment balance at which the automobile stability is still preserved is of the form:

$$M_{пер} = M_{ст} \Rightarrow \frac{G_а}{g} \cdot j_{[\lim]} \cdot h_g = G_а \cdot b. \qquad (34)$$

From the moment balance equation (34), we determine the dependence of the limiting acceleration on the edge of overturning $j_{[\mathrm{lim}]}$ on the gravity centre coordinates:

$$j_{[\mathrm{lim}]} = g \cdot \frac{b}{h_g}. \qquad (35)$$

Considering that:

- the second term in braces of dependence (31) at low speed is insignificant;

- the limiting automobile acceleration on the edge of overturning occurs without touching the driven wheels of the surface, i.e. f_1=0;

then the balance of limiting acceleration and acceleration on the edge of automobile overturning will be of the form:

$$j_{\mathrm{lim}} = j_{[\mathrm{lim}]} \Rightarrow \frac{\varphi_x \cdot \dfrac{a}{L}}{1-\varphi_x \cdot \dfrac{h_g}{L}} \cdot g = g \cdot \frac{b}{h_g}. \qquad (36)$$

After transformations, we get:

$$\varphi_x \cdot \frac{a}{L} = \frac{b}{h_g} \cdot \left(1-\varphi_x \cdot \frac{h_g}{L}\right). \qquad (37)$$

From equation (37), we determine the value of the height coordinate h_g, depending on the longitudinal coordinates of the automobile gravity centre, at which it is possible to reach the physical limit of the limiting automobile acceleration $j_{\varphi\mathrm{lim}}$ preserving longitudinal stability for a given value of the friction coefficient φ_x:

$$h_g = \frac{b}{\varphi_x}. \qquad (38)$$

When (38) is fulfilled, the car will be able to start moving with the physically possible limiting acceleration $j_{\varphi\mathrm{lim}}$ without overturning if the engine can create traction force on the driving wheels more than driving tire-to-surface friction. With an increase in speed, as can be seen from (31), the value of the limiting acceleration decreases and at a certain speed, the power of the air resistance will become so large that the engine power will not be enough to create a traction force on the wheels more than the force of their friction. After reaching this speed, the car will be able to move with maximum acceleration j_{max}, which is limited by the engine power.

The maximum automobile acceleration is determined from the power balance, assuming that all the engine power is used for acceleration:

$$\frac{G_{\mathrm{a}}}{g} \cdot \delta_{\mathrm{вр}} \cdot j_{\mathrm{max}} \cdot v = N_e \cdot \eta_{\mathrm{тр}} - G_{\mathrm{a}} \cdot f \cdot v - \kappa_{\mathrm{в}} \cdot F \cdot v^3, \qquad (39)$$

where $N_e=f(v)$ is engine power at speed v; f is the coefficient of resistance to rolling the wheels of driven and driving axles of the vehicle.

It should be noted that equation (39) takes into account the inertia of the rotating masses because it reduces the traction force on the driving wheels and affects the maximum automobile acceleration. From equation (39), after transformations, we determine the maximum automobile acceleration:

$$j_{\mathrm{max}} = \left[\frac{N_e \cdot \eta_{\mathrm{тр}}}{G_{\mathrm{a}} \cdot v} - f - \frac{\kappa_{\mathrm{в}} \cdot F \cdot v^2}{G_{\mathrm{a}}}\right] \cdot \frac{\delta_{\mathrm{вр}}}{g}. \qquad (40)$$

The value of the speed $v_{j\mathrm{lim}}$, to which the car can accelerate with maximum acceleration, is determined from the condition $j_{\mathrm{lim}}=j_{\mathrm{max}}$, that is, we equate (31) and (40). After transformations, we obtain equation (41):

$$\frac{N_e \cdot \eta_{\mathrm{тр}}}{G_{\mathrm{a}} \cdot \delta_{\mathrm{вр}} \cdot v_{j\mathrm{lim}}} = \frac{(\varphi_x + f_1) \cdot \dfrac{a}{L} - f_1}{1-(\varphi_x + f_1) \cdot \dfrac{h_g}{L}} + \frac{\kappa_{\mathrm{в}} \cdot F \cdot v_{j\mathrm{lim}}^2}{G_{\mathrm{a}}} - \frac{\kappa_{\mathrm{в}} \cdot F \cdot v_{j\mathrm{lim}}^2}{G_{\mathrm{a}} \cdot \delta_{\mathrm{вр}}} + \frac{f}{\delta_{\mathrm{вр}}}. \qquad (41)$$

The difference between the second and third terms in equation (41) is small, so it can be ignored. Taking this assumption into account, using equation (41), we obtain the value of the speed $v_{j\mathrm{lim}}$, to which the engine can provide maximum automobile acceleration (42):

$$v_{j\mathrm{lim}} = \frac{N_e \cdot \eta_{\mathrm{тр}}}{\left[\dfrac{(\varphi_x + f_1) \cdot \dfrac{a}{L} - f_1}{1-(\varphi_x + f_1) \cdot \dfrac{h_g}{L}} \cdot \delta_{\mathrm{вр}} + f\right] \cdot G_{\mathrm{a}}}. \qquad (42)$$

If the gravity centre coordinates of the vehicle and the values of the friction coefficient correspond to (38), then its acceleration j_{lim} occurs without touching the front axle wheels of the surface. As in this case f_1=0, therefore, equation (42) is of the form:

$$v_{j\lim} = \frac{N_e \cdot \eta_{\text{тр}} \cdot \left(1 - \varphi_x \cdot \frac{h_g}{L}\right)}{G_{\text{a}} \cdot \left[\frac{a}{L} \cdot \delta_{\text{вр}} \cdot \varphi_x + \left(1 - \varphi_x \cdot \frac{h_g}{L}\right) \cdot f_2\right]}, \quad (43)$$

where f_2 is the coefficient of resistance to the driving axle wheels of the vehicle rolling.

The engine power N_e=f(v) at a speed of $v_{j\lim}$ in equation (43) is represented as a second order polynomial:

$$N_e = a_1 \cdot v_{j\lim}^2 + a_2 \cdot v_{j\lim} + a_3, \quad (44)$$

where a_1, a_2, a_3 are polynomial coefficients.

Also, in equation (43), we introduce the designation of the ratio characterizing the automobile design factors and the conditions of tire-to-surface friction:

$$A_{\text{a}} = \frac{\eta_{\text{тр}} \cdot \left(1 - \varphi_x \cdot \frac{h_g}{L}\right)}{G_{\text{a}} \cdot \left[\frac{a}{L} \cdot \delta_{\text{вр}} \cdot \varphi_x + \left(1 - \varphi_x \cdot \frac{h_g}{L}\right) \cdot f_2\right]}. \quad (45)$$

From equation (43), taking into account (44) and (45), after transformations, we obtain:

$$v_{j\lim}^2 + \frac{A_{\text{a}} \cdot a_2 - 1}{A_{\text{a}} \cdot a_1} \cdot v_{j\lim} + \frac{a_3}{a_1} = 0. \quad (46)$$

Obviously, the solution to equation (46) has two values:

$$v_{j\lim1,2} = -\frac{A_{\text{a}} \cdot a_2 - 1}{2A_{\text{a}} \cdot a_1} \pm \pm\sqrt{\left(-\frac{A_{\text{a}} \cdot a_2 - 1}{2A_{\text{a}} \cdot a_1}\right)^2 - \frac{a_3}{a_1}}. \quad (47)$$

One value $v_{j\lim1}$ characterizes the speed at which the driving wheels begin to slip due to an increase in traction force higher than the friction force. The second value $v_{j\lim2}$ characterizes the speed at which the driving wheels stop slipping due to a decrease in traction force to the level of friction force.

Theoretical studies of automobile acceleration process at maximum engine power

For theoretical studies of the possibilities of fast passing a given distance by a car, a dragster was chosen. The parameters of dragster are equal to: total mass m_{a}=732 kg; the corresponding mass on the front and rear axles, respectively, m_1=231 kg, m_2=501 kg; car base L = 2.545 m; gravity centre coordinates a=1.742 m and h_g=0.168 m; gearbox ratios 1.857, 1.156, 0.838, 0.683; the differential ratio is 4.111; static wheel radius r_{c}=0.31 m; maximum engine power N_e=220 kW at maximum shaft speed $n_{\max}$=5800 rpm; frontal area F=1.445 m^2; air resistance coefficient $\kappa_{\text{в}}$=0.429 N·c^2/m^4; transmission efficiency $\eta_{\text{тр}}$=0.95; tire-to-surface friction coefficient φ_x=1.2.

The maximum automobile acceleration in the corresponding gears $j_{\text{a}1}$, $j_{\text{a}2}$, $j_{\text{a}3}$, $j_{\text{a}4}$ (see Fig. 2) are determined using the formula (1).

It should be noted that these values coincide with the values of equation (40) for the corresponding speed. Automobile acceleration from stop to speed $v_{\min}$ occurs with the clutch slipping. This area is very small and is not taken into account in this analysis of vehicle acceleration. The limiting automobile acceleration $j_{\lim}$, determined using the formula (31), occurs in the first gear with the driving wheel slipping in the section $v_{\min}$ – $v'_{j\lim}$, that is, up to the maximum engine speed.

After switching from the first gear (point c') to the second one (point d), the vehicle accelerates without driving wheel slipping. The maximum acceleration values when speeding up in the respective gears are marked with full lines. In this case, the automobile acceleration characteristic is represented by the curve a in Figure 2. The time of passing a section of 400 meters is equal to 10.7 s.

If we ensure the growth of the limiting acceleration $j_{\lim}$ up to the absolute physical acceleration limit $j_{\text{ф}\lim}$, then it is possible to reduce the time of overcoming the given distance. To do this, it is enough to change the height of the automobile gravity centre without any other changes in accordance with the condition (38).

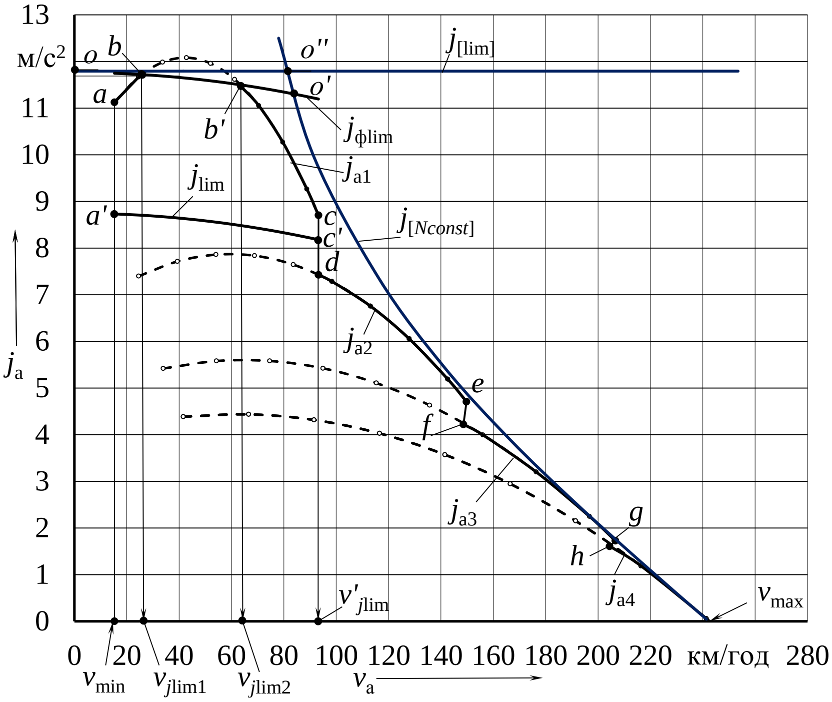

Fig. 2. Automobile acceleration when speeding up: j_{a1}, j_{a2}, j_{a3}, j_{a4} are limiting acceleration in the corresponding gears; j_{lim} is limiting acceleration; $j_{\phi lim}$ is an absolute physical limit of limiting acceleration; $j_{[lim]}$ limiting acceleration under the condition of longitudinal stability; $j_{[Nconst]}$ is the maximum acceleration at constant power on the driving wheels

In this case, the limiting acceleration j_{lim} acquires the value of absolute physical acceleration limit $j_{\phi lim}$, and the vehicle accelerates in the first gear with speeding up indicated by *a-b-b'-c* curve. In this case, the slipping of the driving wheels occurs from the speed v_{jlim1} to the speed v_{jlim2}, and the subsequent acceleration in the first gear occurs without their slipping up to the speed, which is limited by the maximum rotational speed of the engine shaft (point *c*). Dependence *b* (Fig. 2) represents the time to cover the distance with an increased value of the automobile gravity centre. Obviously, in this case, the time to cover the distance $[S_p]$=400 m is reduced to 10.2 s.

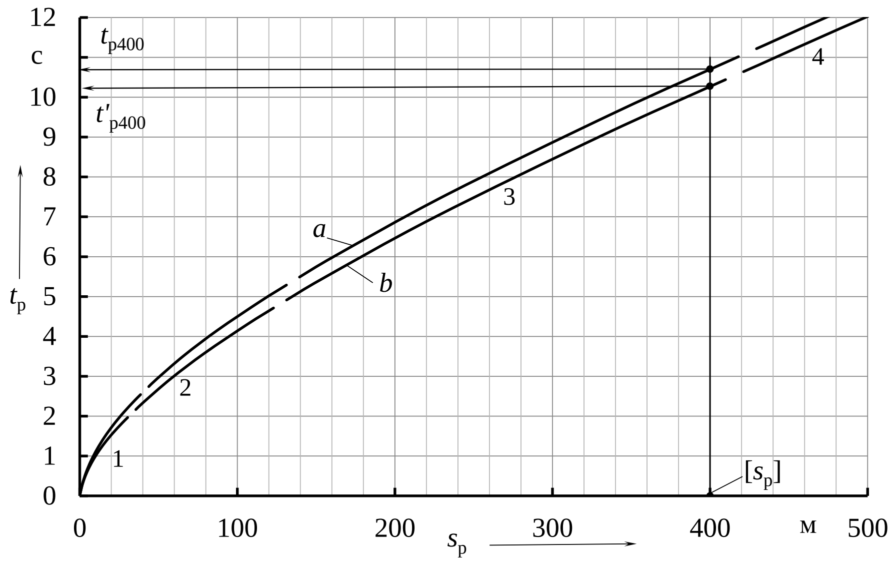

Fig. 3. Automobile acceleration characteristics: *a* at h_g=1.168 m and *b* at h_g=1.168 m; t'_{p400}, t_{p400} – time to cover a distance of 400 m; 1, 2, 3, 4 – the number of a gear on which acceleration occurs

If the car is equipped with an engine and a transmission that provide the driving wheel power $N_к$ regardless of the driving speed, then its maximum accelerations depending on the driving speed are displayed by the curve $j_{[Nconst]}$. The value of the speed $v_{j\lim}$, calculated by the formula (5), will correspond to the value of the point *o*', respectively, if we take into account that at a given speed, the inertia of the automobile rotating masses is internal. In this case, the vehicle can move with acceleration $j_{\phi\lim}$, from point *o* to point *o*' and furthermore with acceleration $j_{[Nconst]}$. This will allow us to further reduce the time for covering the distance.

Conclusion

On the basis of a comprehensive analysis of the tire-to-surface friction process and air resistance when accelerating at maximum engine power, the dependences of automobile acceleration on its design factors and the driving speed were obtained. As a result of theoretical studies of automobile acceleration at maximum engine power, the optimal value of the height of the automobile gravity centre, at which the absolute physical limit of its acceleration $j_{\phi\lim}$ is reached while maintaining its longitudinal stability on the surface with a given friction coefficient. The obtained dependences can be used in designing new and improving racing cars like dragsters.

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Conflict of interests

The authors declare no conflict of interest regarding the publication of this article.

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Граничне прискорення автомобіля

Анотація. Проблема. *Недоліком існуючих залежностей визначення показників розгону при максимальному використанні потужності двигуна і показників взаємодії ведучих коліс з опорною поверхнею є те, що вони є коректними тільки якщо параметри двигуна та трансмісії є такими, що забезпечують підведення потужності до ведучих коліс, які не буксують, незалежно від швидкості руху. Щоб усунути зазначений недолік необхідно враховувати, що потужність яка підводиться до ведучих коліс залежить від частоти обертання валу двигуна, і значить від швидкості руху при розгоні автомобіля.* ***Мета.*** *Мета роботи - подальший розвиток теорії автомобіля шляхом удосконалення залежностей, які дозволяють визначати показники розгону автомобіля і встановити характер процесу його розгону від конструктивних параметрів.* ***Методологія.*** *Підходи, прийняті в роботі для досягнення цієї мети базуються на законах фізики, теоретичної механіки і положеннях теорії автомобіля.* ***Результати.*** *Удосконалено аналітичні залежності для визначення максимального і граничного прискорення автомобіля при розгоні в залежності від його конструктивних параметрів і швидкості руху. Отримано залежності для визначення діапазону буксування ведучих коліс від швидкості руху при розгоні автомобіля і граничного прискорення автомобіля за умовою його поздовжньої стійкості. При теоретичному дослідженні процесу розгону автомобіля встановлено, що розроблені залежності дозволяють визначити характер руху автомобіля і оцінити вплив на показники розгону його конструктивних параметрів.* ***Оригінальність.*** *Отримані залежності для визначення максимального і граничного прискорення, діапазону швидкостей руху з буксуванням коліс при розгоні автомобіля дозволили уточнити уявлення про характер руху при розгоні і вплив конструктивних параметрів автомобіля на показники розгону.* ***Практичне значення.*** *Отримані залежності можуть бути використані при проектуванні нових і при удосконаленні спортивних автомобілів типу дрегстер, та для аналізу динаміки руху автомобіля при його розгоні з повної подачею палива та визначення характеру взаємодії ведучих коліс з опорної поверхнею в залежності від швидкості руху.*

Ключові слова: *рекордно-гоночний автомобіль, прискорення, граничне прискорення, коефіцієнт зчеплення, координати центру тяжіння, потужність, розгінна характеристика.*

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