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Two inverse non-stationary problems of axially symmetric deformation of a finite-length elastic cylindrical shell

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Annotation. Problem. *Among the many problems of the solid mechanics, there is a whole class of problems that are related to inverse problems. In turn, among the inverse problems, many problems are ill-posed. Obtaining an exact analytical solution of such problems is related to certain mathematical difficulties and requires using special methods.* ***Goal.*** *The goal of the study is to obtain analytical solutions for inverse problems of the identification of non-stationary load and the control of non-stationary vibrations of a cylindrical shell with asymmetric boundary conditions.* ***Methodology.*** *In this investigation, a refined theory of medium-thickness shells was used. Fourier series expansion, the theory of integral equations and the Laplace transform were used to obtain the solution of the direct problem. Tikhonov's regularization method was used to solve inverse problems.* ***Results.*** *As a result of the investigation, the solutions of two inverse problems of the solid mechanics were obtained. The first task is to identify a fixed and moving concentrated axisymmetric non-stationary force acting on a cylindrical shell, based on the displacement values at any point of the shell; identification of two fixed concentrated forces. The second task is to control vibrations at any point of the cylindrical shell by introducing an auxiliary concentrated force. Numerical results obtained demonstrate the fulfillment of the control criterion as a result of the action of the given and auxiliary force.* ***Originality.*** *Analytical solutions of the inverse problems of the solid mechanics for a cylindrical shell of medium thickness with asymmetric boundary support conditions are obtained.* ***Practical value.*** *The technique received allows effective identification of an unknown non-stationary load. It's important for the rational design of reliable cylindrical shell structures. Its use also makes possible to create a theoretical basis to control the deflected mode parameters of cylindrical shell structural elements.*

Key words: *cylindrical shell, non-stationary load, inverse problem, regularization method, identification, vibration control.*

Introduction

Identification of external loads and controlling the vibrations of constructional elements can be related to inverse problems in mechanics of a deformed solid. The complexity of their solution consists in that these problems are frequently ill-posed.

Many papers deal with calculating of the deflected mode parameters in constructional elements under non-stationary load conditions, provided the acting loads are known. Less attention is paid to loads identification problems and vibration control. After mathematical physics methods for solving inverse problems came into being, there appeared the possibility of applying them to solving non-stationary problems in identification and control of the deflected mode of constructional elements, which undergo non-stationary deformation.

Analysis of publications

Modeling of processes that occur in constructional elements under load applied is based on the application of the theory of vibrations. A wide review of the models used to describe the vibrations such constructional elements as rods, plates and shells is given in [1]. Their use makes it possible to solve a number of problems in the solid mechanics, including specific ones.

For example, in [2] the equilibrium equations of a rib grid-stiffened composite cylindrical shell reinforced with carbon nanotubes were obtained and the effects of grid ribs on the dynamic response of the shell were studied. The investigation the transient responses of cylindrical shells induced by moving and simultaneous impulsive loads were carried out in [3]. The problem of the forced vibrations of a discretely reinforced cylindrical shell on an elastic foundation under impulsive loading is described in [4-5]. The problem of minimizing the mass of layered orthotropic constant-thickness non-closed shells at impulse loading was solved in [6].

Inverse problems should be considered as a special class of problems in the solid mechanics. Their solutions allow obtaining important results. For example, in [7], solutions for new non-stationary inverse problems for elastic rods were obtained. The solution of inverse retrospective problems with a completely unknown space-time law of load distribution is based on the method of influence functions. The inverse problem to predict buckling of a cylindrical shell under an external pressure is considered in [8]. The work [9] presents an inverse problem to predict dynamic loads applied to the conical shells using the finite difference method.

When solving inverse problems in the solid mechanics, regularization methods proved to be quite effective [10-12]. One of them is the Tikhonov's regularization method [13, 14]. It widely uses to solve different inverse problems in the solid mechanics.

In [15] based on augmented Tikhonov's regularization method, a new computational inverse method is proposed to reconstruct impact loads acting on composite laminated cylindrical shell with random characteristics. In [16], Tikhonov's regularization method was used to determine the dynamic load in a mechanical system with four degrees of freedom. The problem to control non-stationary vibrations of a rectangular plate by introducing an additional (controlling) load is considered in [17]. The problem is solved using the no classical theory of plates and Tikhonov's regularization method. The paper [18] presents a solving the problem of controlling non-stationary vibrations at a certain point of a rectangular plate by introducing an auxiliary load, the law of change in time of which is to be determined. Identification of non-stationary loads acting to a simply supported shell supported by concentric stiffeners is considered in [19]. In the article [20], using the Tikhonov's regularization method, inverse problems are solved for a number of different constructional elements in the form of plates and shells.

Based on the analysis carried out, it can be concluded that the solving of inverse problems for constructional elements in the solid mechanics is relevant. At the same time, the obtaining exact analytical solutions of inverse problems for specific constructional elements under non-stationary loading are insufficiently studied.

Purpose and Tasks

The goal of the investigation is to obtain analytical solutions for inverse problems of the identification non-stationary load and the control non-stationary vibrations of a cylindrical shell with asymmetric boundary conditions.

To achieve this goal, the following tasks were set:

– development of a mathematical model to determine the deflected mode parameters of a cylindrical shell under non-stationary load acting;

– solving the inverse problem to identify an non-stationary load acting on a cylindrical shell with asymmetric boundary conditions;

– solving the problem of controlling vibrations for cylindrical shell with asymmetric boundary conditions.

Direct problem

We shall consider a closed circular cylindrical shell with the following boundary conditions: the left edge of the shell is simply supported with slippage along the axis of the shell, and the right edge is clamped with slippage along the axis of the shell (Fig. 1). The shell is subject to action of a normal non-stationary concentrated force in point x_p.

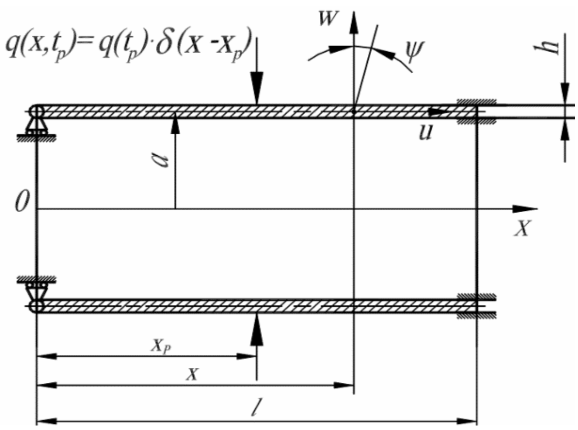


Fig 1. Investigated mechanical system

This problem is solved by employing a technique based on introducing an additional compensating concentrated moment $M_0(t)$, which ensures absence of the shell normal rotation angle at the right edge of the shell.

The response of an average-thickness shell of the Timoshenko theory type to axially symmetric transverse and concentrated moment loads is simulated by a system of linear differential equations [8]:

$$\frac{\partial^2 u}{\partial \xi^2}+\frac{\nu l}{a}\frac{\partial w}{\partial \xi}-\frac{\partial^2 u}{\partial t^2}=0;$$
$$\bar{k}^2\left(\frac{\partial^2 w}{\partial \xi^2}+l\frac{\partial \psi}{\partial \xi}\right)-\frac{1}{a}\left(\frac{l^2}{a}w+\nu l\frac{\partial u}{\partial \xi}\right)-$$
$$-\frac{\partial^2 w}{\partial t^2}=\frac{\left(1-\nu^2\right)l^2}{Eh}q\left(\xi,t\right); \tag{1}$$
$$\frac{h^2}{12}\frac{\partial^2 \psi}{\partial \xi^2}-\bar{k}^2\left(l\frac{\partial w}{\partial \xi}+\psi l^2\right)-\frac{h^2}{12}\frac{\partial^2 \psi}{\partial t^2}=$$
$$=\frac{12\left(1-\nu^2\right)l^2}{Eh^3}M_0\left(t\right)\frac{\delta(1-\xi)}{l},$$

where $\xi=\frac{x}{l}$; $t=\frac{t_p\sqrt{E}}{l\sqrt{\rho\left(1-\nu^2\right)}}$; $\bar{k}^2=\frac{1-\nu}{2}k^2$; t is dimensionless time; t_p is dimensional time; u and w are displacements of points on the median surface in axial and radial directions respectively; ψ is rotation angle of the normal with respect to the median surface of the shell; k is shear coefficient; $M_0(t)$ is compensating moment; and $q(\xi,t)$ is the transverse load.

The boundary conditions for the mechanical system considered (Fig. 1) have the form:

$$\begin{aligned}&N_x(\xi,t)\big|_{\substack{\xi=0\\ \xi=1}}=0; \quad w(\xi,t)\big|_{\substack{\xi=0\\ \xi=1}}=0;\\ &M_x(\xi,t)\big|_{\xi=0}=0; \quad \psi(\xi,t)\big|_{\xi=1}=0.\end{aligned} \tag{2}$$

The solution of a problem with boundary conditions of form (2) is reduced to the problem of non-stationary vibration of a simply-supported shell.

Substituting right edge clamping of the cylindrical shell with a simply-supported one with a compensating moment allows searching for the required functions (displacements and rotation angle of the normal) in the form of their expansion into the following trigonometric Fourier series:

$$\begin{aligned}&w\left(\xi,t\right)=\sum_{k=1}^{\infty}a_k\left(t\right)\sin\left(k\pi\xi\right);\\ &u\left(\xi,t\right)=\sum_{k=1}^{\infty}b_k\left(t\right)\cos\left(k\pi\xi\right);\\ &\psi\left(\xi,t\right)=\sum_{k=1}^{\infty}c_k\left(t\right)\cos\left(k\pi\xi\right),\end{aligned} \tag{3}$$

where $a_k(t)$, $b_k(t)$, and $c_k(t)$ are unknown expansion coefficients.

Expansion coefficients (3) are found from (1) by using the properties of orthogonality of Fourier series, and by applying the Laplace transform. The functions for defining $w(\xi,t)$, $u(\xi,t)$ and $\psi(\xi,t)$ have the form:

$$w(\xi,t)=\sum_{n=1}^{\infty}\left[\sum_{r=1}^{3}\frac{-I_{nr}(t)\left(C_n-\alpha_{nr}^2\right)\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}+\sum_{r=1}^{3}\frac{l\pi n\bar{k}^2\cdot T_{nr}(t)\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\right]\sin\left(n\pi\xi\right);$$

$$u(\xi,t)=\sum_{n=1}^{\infty}\left[\sum_{r=1}^{3}\frac{-I_{nr}(t)\frac{\nu l\pi n}{a}\left(C_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}+\sum_{r=1}^{3}\frac{T_{nr}(t)\cdot\frac{\nu l^2\pi^2n^2\bar{k}^2}{a}}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\right]\cos\left(n\pi\xi\right); \tag{4}$$

$$\psi(\xi,t)=\sum_{n=1}^{\infty}\left[\sum_{r=1}^{3}\frac{\frac{12}{h^2}I_{nr}(t)l\pi n\bar{k}^2\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}+\sum_{r=1}^{3}\frac{T_{nr}(t)\left(\frac{\nu^2l^2\pi^2n^2}{a^2}-\left(A_n-\alpha_{nr}^2\right)\left(B_n-\alpha_{nr}^2\right)\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\right]\cos\left(n\pi\xi\right),$$

where: $I_{nr}\left(t\right)=\frac{2\left(1-\nu^2\right)l^2}{Eh}\int\limits_0^t Q_n\left(\xi,\tau\right)\sin\alpha_{nr}\left(t-\tau\right)d\tau$; $T_{nr}\left(t\right)=\frac{288\left(1-\nu^2\right)l^2}{Eh^5}\int\limits_0^t M_0\left(\tau\right)\frac{\cos\left(n\pi\right)}{l}\sin\alpha_{nr}\left(t-\tau\right)d\tau$;

$$A_n = \bar{k}^2 n^2 \pi^2 + \frac{l^2}{a^2}; \quad C_n = n^2 \pi^2 + 12\frac{\bar{k}^2 l^2}{h^2};$$

$$B_n = n^2 \pi^2; \; Q_n(\xi, \tau) = \int_0^1 q(\xi, \tau) \sin(n\pi\xi)\, d\xi;$$

where α_{nr} are natural vibration frequencies of the simply-supported shell.

The time component of the concentrated moment load (function $M_0(t)$) is defined by using the corresponding Volterra integral equation of 1-st kind following from condition $\psi(1,t)=0$.

Inverse problem

Identifying a normal concentrated non-stationary load acting on a cylindrical shell consists in determining the law of variation in time of a load by defining flexure values w in a certain point ξ_0 of the cylindrical shell as a known time function $f(t)$.

Taking into account also the specified condition of equality to zero of the rotation angle of a normal at the shell edge in the clamping, one can form a system of 1-st kind Volterra integral equations for $q(t)$ and $M_0(t)$:

$$w(\xi_0, t) = f(t); \quad \psi(1,t) = 0. \tag{5}$$

For a numerical solution of system (5), the Duhamel integrals included therein are approximated by finite sums based on the formulas of rectangles.

$$I_{nrm} = \frac{2(1-\nu^2) l^2}{Eh} \sum_{p=1}^{m} q_p \cdot \frac{\sin(n\pi\xi_P)}{l} \times$$
$$\times \frac{\cos[\alpha_{nr}(m-p)\Delta t] - \cos[\alpha_{nr}(m-p+1)\Delta t]}{\alpha_{nr}};$$

(6)

$$T_{nrm} = \frac{288(1-\nu^2) l^2}{Eh^5} \sum_{p=1}^{m} M_{0p} \cdot \frac{\cos(n\pi)}{l} \times$$
$$\times \frac{\cos[\alpha_{nr}(m-p)\Delta t] - \cos[\alpha_{nr}(m-p+1)\Delta t]}{\alpha_{nr}},$$

where $m{\cdot}\Delta t$ is the time interval considered; $m=0,1, \ldots, M$ is number of time intervals; ξ_p is the value of the dimensionless axial coordinate of the point to which the concentrated load is applied. The time step is designated as Δt.

To find the unknown loads, it is convenient to present system (5) in matrix form:

$$\begin{aligned} &\mathrm{B}_1 \cdot \mathrm{q} + \mathrm{C}_1 \cdot \mathrm{M}_0 = \mathrm{w}; \\ &\mathrm{D} \cdot \mathrm{q} + \mathrm{E} \cdot \mathrm{M}_0 = 0, \end{aligned} \tag{7}$$

where $\mathbf{q}$, $\mathbf{M_0}$ and $\mathbf{w}$ are column vectors corresponding to functions $q(t)$, $M_0(t)$ and $w(t)$; $\mathbf{B_1}$, $\mathbf{C_1}$, $\mathbf{D}$, and $\mathbf{E}$ are matrices. The elements of matrices are defined as follows:

$$B_{1m,p} = \sum_{n=1}^{\infty}\sum_{r=1}^{3} \frac{-2(1-\nu^2) l^2}{Eh} \frac{(C_n - \alpha_{nr}^2)(B_n - \alpha_{nr}^2)}{\alpha_{nr} \prod_{\substack{j=1 \\ j\neq r}}^{3} (\alpha_{nj}^2 - \alpha_{nr}^2)} \frac{\sin(n\pi\xi_P)}{l} k_{nrmp} \sin(n\pi\xi_0);$$

$$C_{1m,p} = \sum_{n=1}^{\infty}\sum_{r=1}^{3} \frac{288(1-\nu^2) l^2}{Eh^5} \frac{l\pi n \bar{k}^2 (B_n - \alpha_{nr}^2)}{\alpha_{nr} \prod_{\substack{j=1 \\ j\neq r}}^{3} (\alpha_{nj}^2 - \alpha_{nr}^2)} \frac{\cos(n\pi)}{l} k_{nrmp} \sin(n\pi\xi_0);$$

$$D_{m,p} = \sum_{n=1}^{\infty}\sum_{r=1}^{3} \frac{2(1-\nu^2) l^2}{Eh} \frac{\frac{12 l\pi n \bar{k}^2}{h^2} (B_n - \alpha_{nr}^2)}{\alpha_{nr} \prod_{\substack{j=1 \\ j\neq r}}^{3} (\alpha_{nj}^2 - \alpha_{nr}^2)} \frac{\sin(n\pi\xi_P)}{l} k_{nrmp} \cos(n\pi); \tag{8}$$

$$E_{m,p} = \sum_{n=1}^{\infty}\sum_{r=1}^{3} \frac{288(1-\nu^2) l^2}{Eh^5} \frac{\left[\frac{\nu^2 l^2 \pi^2 n^2}{a^2} - (A_n - \alpha_{nr}^2)(B_n - \alpha_{nr}^2)\right]}{\alpha_{nr} \prod_{\substack{j=1 \\ j\neq r}}^{3} (\alpha_{nj}^2 - \alpha_{nr}^2)} \frac{\cos(n\pi)}{l} k_{nrmp} \cos(n\pi),$$

where $k_{nrmp} = \dfrac{\cos[\alpha_{nr}(m-p)\Delta t] - \cos[\alpha_{nr}(m-p+1)\Delta t]}{\alpha_{nr}}$.

After eliminating $\mathbf{M_0}$ from (7), we obtain the matrix equation for the time component of the identified load:

$$A \cdot q = w, \qquad (9)$$

where matrix $A = B_1 - C_1 \cdot E^{-1} \cdot D$. Relationship (9) is the matrix analog of the Volterra integral equation of 1-st kind. To derive an approximate and steady solution of this equation, it is necessary to apply Tikhonov's regularization method [1, 2].

The solution of matrix equation (9) is reduced to solving a regularized system of linear algebraic equations (SLAE) of the type:

$$\left(A^T \cdot A + \alpha \cdot C\right) q = A^T \cdot w, \qquad (10)$$

where $\mathbf{A^T}$ is a transposed matrix with respect to matrix $\mathbf{A}$; $\mathbf{C}$ is a symmetric three-diagonal matrix whose form is given in [1].

Equation (10) includes regularization parameter α whose value is selected according to the residual principle, viz. coordinating the value of the residual on a regularized solution with account of the error in the right-hand part of the initial SLAE.

The method offered also allows to identify moving loads of the form $q\left(\xi, t_p\right) = q\left(t_p\right) \dfrac{\delta\left(\xi - V_0 t\right)}{l}$, where V_0 is the specified value of the velocity of concentrated force movement. Note that the methods for calculating the impact of moving loads on elasto-deformed elements of constructions, including inertia ones, which are based on solving direct problems, are described in monograph [9].

The suggested method of defining the time dependence of one concentrated load can be generalized to determine the variation in time of two and more concentrated loads.

Figure 2 can be considered as a loading scheme of a shell with two concentrated loads, where the laws of variation in time of loads acting on a shell, defined by functions $q_1(t)$ and $q_2(t)$, are unknown.

The relationships for identifying two concentrated loads by displacement values w, specified in two points of the shell, can be presented in matrix form as:

$$\begin{aligned} &B_{11} \cdot q_1 + B_{12} \cdot q_2 + C_{11} \cdot M_0 = w_1; \\ &B_{21} \cdot q_1 + B_{22} \cdot q_2 + C_{12} \cdot M_0 = w_2; \\ &D_1 \cdot q_1 + D_2 \cdot q_2 + E \cdot M_0 = 0. \end{aligned} \qquad (11)$$

In equations (11), matrices $\mathbf{B_{11}}$, $\mathbf{B_{12}}$, $\mathbf{B_{21}}$, $\mathbf{B_{22}}$, $\mathbf{C_{11}}$, $\mathbf{C_{12}}$, $\mathbf{D_1}$ and $\mathbf{D_2}$ are derived from matrices $\mathbf{B_1}$, $\mathbf{C_1}$ and $\mathbf{D}$ by introducing corresponding coordinates of displacement registration points and points of application of loads (for example, matrix $\mathbf{B_{12}}$ corresponds to matrix $\mathbf{B_1}$ with the displacement registration point coordinate $\mathbf{w_1}$, and the load application point coordinate $\mathbf{q_2}$; matrix $\mathbf{C_{12}}$ answers matrix $\mathbf{C_1}$ with displacement registration point coordinate $\mathbf{w_2}$; matrix $\mathbf{D_2}$ is derived on the basis of matrix $\mathbf{D}$ with account of the load application point coordinate $\mathbf{q_2}$).

The values of two concentrated loads can be obtained by presenting the solution of the system of matrix equations (11) as:

$$A \cdot q_1 = f_1; \;\; A \cdot q_2 = f_2, \qquad (12)$$

where the following designations of matrices and column vectors are used:

$$\begin{gathered} A = B_{11} \cdot \left(B_{22} \cdot E - C_{12} \cdot D_2\right) - B_{12} \times \\ \times \left(B_{21} \cdot E - C_{12} \cdot D_1\right) + C_{11} \cdot \left(B_{21} \cdot D_2 - B_{22} \cdot D_1\right); \end{gathered}$$

$$\begin{gathered} f_1 = \left(B_{22} \cdot E - C_{12} \cdot D_2\right) \cdot w_1 - B_{12} \cdot E \cdot w_2 + \\ + C_{11} \cdot D_2 \cdot w_2; f_2 = B_{11} \cdot E \cdot w_2 - \\ -\left(B_{21} \cdot E - C_{12} \cdot D_1\right) \cdot w_1 - C_{11} \cdot D_1 \cdot w_2. \end{gathered}$$

A system of two loads is identified by applying Tikhonov's regularization procedure to each of matrix equations (12).

Controlling problem

Let us consider the problem of controlling cylindrical shell vibrations.

The cylindrical shell is acted upon with transverse non-stationary axially symmetric concentrated load $q_1(t)$ whose law of variation in time is known. As a result of its action, a deformation process occurs in the cylindrical shell, which causes its non-stationary vibration. The objective of the controlling problem, at non-stationary vibrations of the cylindrical shell, is to define controlling load $q_2(t)$ (applied in point ξ_2) whose combined action with load $q_1(t)$ (applied in point ξ_1) would ensure fulfilment of the required controlling criterion (a specified law of variation in time of displacement in certain point ξ_0 (controlling criterion)). The loading scheme of the shell is shown in Figure 2.

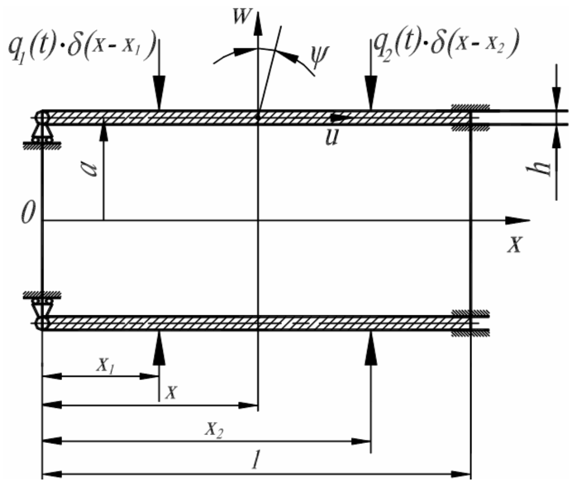


Fig. 2. Loading scheme for the controlling problem

Knowing the flexure vs. time function *f(t)*, and taking into account equality to zero of the rotation angle of the shell edge in the clamping, we have a set of equations:

$$\begin{aligned} w(\xi_0,t) &= f(t); \\ \psi(l,t) &= 0. \end{aligned} \tag{13}$$

Similarly, to the algorithm described earlier in inverse problem, system (13) is reduced to the matrix form:

$$\begin{aligned} \mathrm{B}_1\cdot\mathrm{q}_1+\mathrm{B}_2\cdot\mathrm{q}_2+\mathrm{C}_1\cdot\mathrm{M}_0 &= \mathrm{w}; \\ \mathrm{D}_1\cdot\mathrm{q}_1+\mathrm{D}_2\cdot\mathrm{q}_2+\mathrm{E}_1\cdot\mathrm{M}_0 &= 0, \end{aligned} \tag{14}$$

where $\mathbf{q}_1$ and w are known time functions (load and flexure variation in time, which meet the controlling criterion) approximated as column vectors; $\mathbf{q}_2$ and $\mathbf{M}_0$ are column vectors of unknown time functions; $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}_1$, $\mathbf{D}_1$, $\mathbf{D}_2$ and $\mathbf{E}_1$ are matrices. The elements of matrices $\mathbf{C}_1$ and $\mathbf{E}_1$ can be obtained according to (8), and $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{D}_1$ and $\mathbf{D}_2$ are obtained as follows:

$$\begin{aligned}
B_{1m,p} &= \sum_{n=1}^{\infty}\sum_{r=1}^{3}\frac{-2\left(1-\nu^2\right)l^2}{Eh}\frac{\left(C_n-\alpha_{nr}^2\right)\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\frac{\sin\left(n\pi\xi_1\right)}{l}k_{nrmp}\sin\left(n\pi\xi_0\right); \\
B_{2m,p} &= \sum_{n=1}^{\infty}\sum_{r=1}^{3}\frac{-2\left(1-\nu^2\right)l^2}{Eh}\frac{\left(C_n-\alpha_{nr}^2\right)\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\frac{\sin\left(n\pi\xi_2\right)}{l}k_{nrmp}\sin\left(n\pi\xi_0\right); \\
D_{1m,p} &= \sum_{n=1}^{\infty}\sum_{r=1}^{3}\frac{24\left(1-\nu^2\right)l^3\pi n\bar{k}^{-2}}{Eh^3}\frac{\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\frac{\sin\left(n\pi\xi_1\right)}{l}k_{nrmp}\cos\left(n\pi\right); \\
D_{2m,p} &= \sum_{n=1}^{\infty}\sum_{r=1}^{3}\frac{24\left(1-\nu^2\right)l^3\pi n\bar{k}^{-2}}{Eh^3}\frac{\left(B_n-\alpha_{nr}^2\right)}{\alpha_{nr}\prod\limits_{\substack{j=1\\ j\neq r}}^{3}\left(\alpha_{nj}^2-\alpha_{nr}^2\right)}\frac{\sin\left(n\pi\xi_2\right)}{l}k_{nrmp}\cos\left(n\pi\right).
\end{aligned} \tag{15}$$

The solution of equation (14) is reduced to (9), and then Tikhonov's regularization algorithm is applied.

Numerical results

The cylindrical shell with the following parameters has been considered for numerical analysis: l=1.5 m, a=0.3 m, h=0.043 m, E=2.1·10^{11} Pa, ν=0.3, and k=0.833. The maximum value of load q_0 =10^5 H/m^2, and the duration of load action ω=0.00046 s.

The results of identifying the concentrated load described by a difference of Heaviside's functions as a step-function (16) are shown in Figure 3.

$$q(\xi,t)=q_0\left(H(t)-H(t-\omega)\right)\frac{\delta\left(\xi-\xi_0\right)}{l} \tag{16}$$

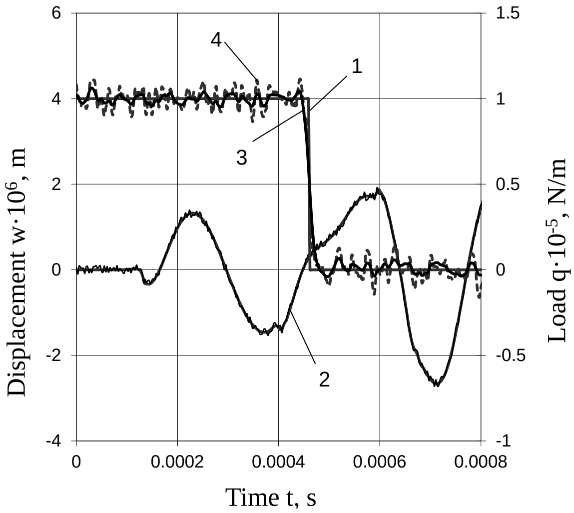


Fig. 3. Identification of the concentrated load

In the Figure, the following curves are designated: 1 is a non-stationary load which was selected as input data when solving the direct problem; 2 is flexure of the shell in a point with coordinate ξ=0.75, which occurs due to action of the non-stationary load mentioned (the flexure curve is also superimposed with a flexure curve with "noise" that models inaccuracies, for instance, of experimental data, which reached 5 % of the maximum flexure amplitude); 3 is the load identified by "noise" data with optimal parameter of regularization α; 4 is the load calculated with the regularization parameter whose value is near to optimum, but still in an area with steady approximated solutions.

The optimal value of the regularization parameter was selected by applying the discrepancy principle [1, 2] in such a manner that the value of the difference of norms $\left\|w^{\delta} - Aq^{\alpha}\right\|_{l_2}^{2} - \left\|\delta\right\|_{l_2}^{2}$ was equal to zero. The graph illustrating how the optimal value of the regularization parameter was selected for a concentrated load is shown in Fig. 4.

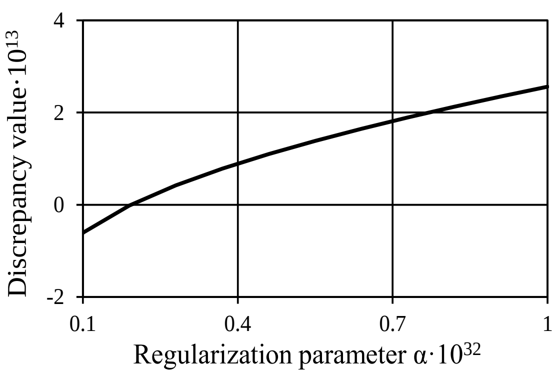


Fig. 4. Selecting the optimal value of the regularization parameter

The graph in Fig. 5 illustrates the effectiveness of the algorithm of selecting an optimal regularization parameter.

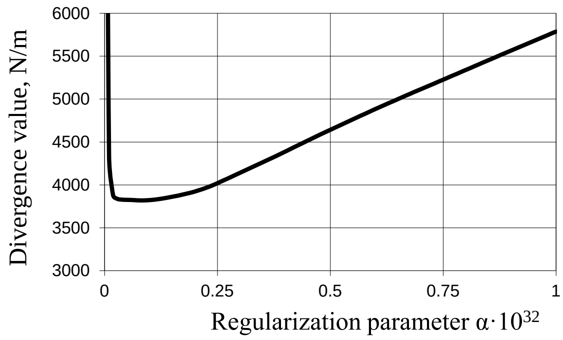


Fig. 5. Divergence value under different regularization parameters

The divergence value is meant to be a parameter which can be named as the average divergence of the values of the load being identified from its exact values. This parameter is calculated by formula $\sum_{m=1}^{M}\left|q_m - q_m{*}\right|/\mathrm{M}$, where M is number of points in which investigated time function q is calculated. Fig. 5 shows the functional relationship between the divergence of identified concentrated load q and load $q*$, which was used as input data for solving the direct problem depending on the value of the regularization parameter. As seen in Figs. 4 and 5, the minimum divergence of values of the identified load is reached near to the optimal value of the regularization parameter defined by the discrepancy (see Fig. 3). The absolute value of the minimum divergence is approximately equal to 3,700 N, this being 3.7 % of the maximum amplitude of the external load. The absolute value of the average divergence, corresponding to an optimal regularization parameter, is approximately equal to 3,800 N, which is 3.8 % of the maximum amplitude of the external load.

The results of identification of a moving annular concentrated load are presented in Fig. 6. It was assumed that the load appears in an initial point of time at the left edge of the shell. Then it moves with a constant velocity of 2,223 m/s to the right edge, and disappears when it reaches the right edge of the cylindrical shell. Curve 1 is a non-stationary moving load; 2 is displacement in point ξ_0=0.75; and 3 is the load that has been identified.

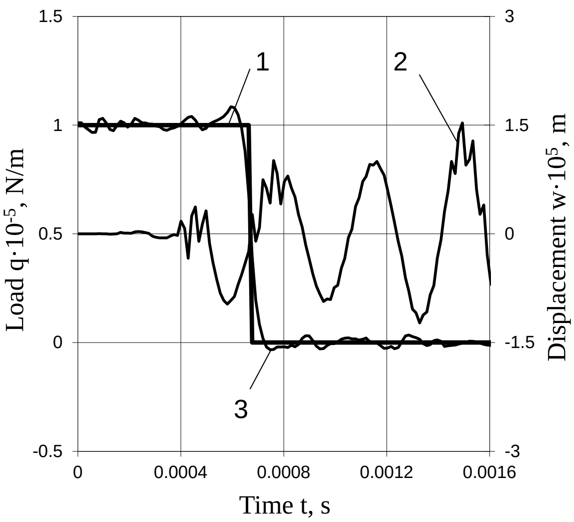
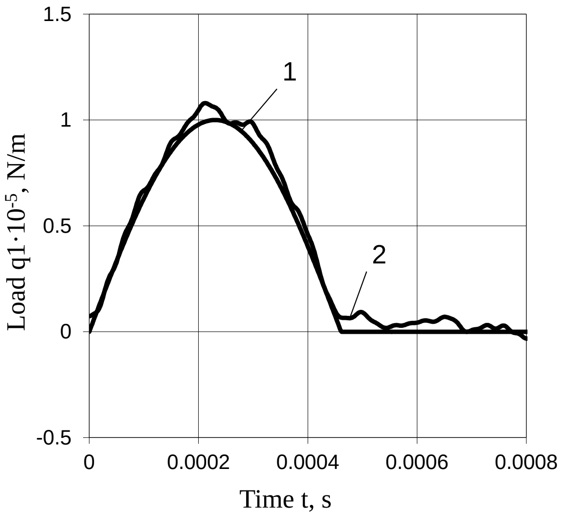
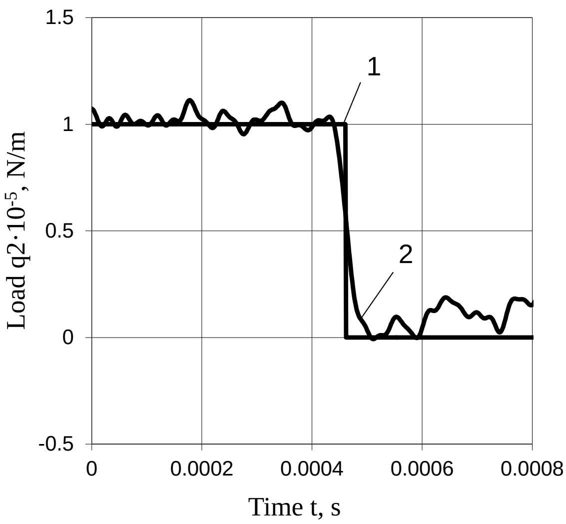


Fig. 6. Identification of a moving concentrated load



a



b

Fig. 7. Identification of two concentrated loads

The results of identifying two concentrated loads with respect to displacement values **w** are presented in Figs. 7 and 8 (curves 2). Fig. 7, a and b, illustrate the variation in time of loads $\mathbf{q_1}$ and $\mathbf{q_2}$ respectively (curves 1), which cause cylindrical shell displacements $\mathbf{w_1}$ and $\mathbf{w_2}$ (Fig. 8, curves 1 and 2 respectively). The loading (variation in time of loads $\mathbf{q_1}$ and $\mathbf{q_2}$), which acts on the cylindrical shell, is identified by using the values of displacements with superimposed "noise" that models initial data errors. The coordinates of the points of application of loads are: $\mathbf{q_1}$ - ξ=0.5; and $\mathbf{q_2}$ - ξ=0.75. The coordinates of points in which displacements are registered are: $\mathbf{w_1}$ - ξ=0.4; and $\mathbf{w_2}$ - ξ=0.9. The input data "noise" level was 5% of the maximum displacement amplitude.

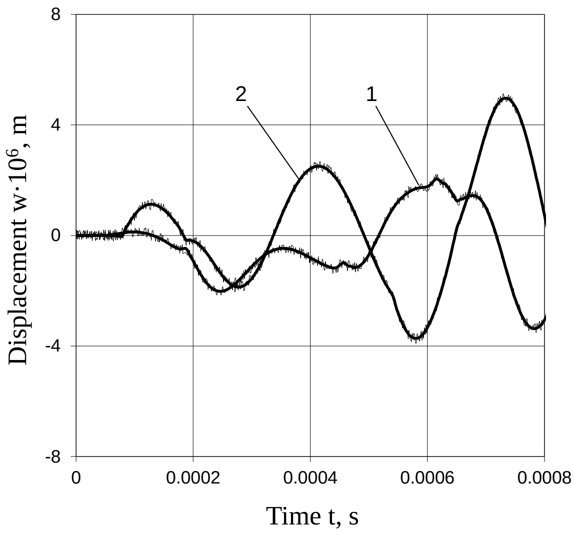


Fig. 8. Input data for identification of two concentrated loads

Fig. 9 shows results related to the problem of controlling the deflected mode of a cylindrical shell under action of an external normal concentrated load.

Flexure $\omega(t)$ in the point with coordinate ξ_0=0.5 (in the middle of the shell) is taken as the control criterion.

The graphs of the controlling load (curve 3) and the external load $q_1(t)$ (curve 1), which change as a sinusoidal half-wave (17), are shown in Figure 9. Curve 4 in this Figure meets the specified controlling criterion. The points of application of loads are: ξ_1=0.35; and ξ_2=0.65.

$$q_1(t) = q_0\left(H(t) - H(t-\omega)\right)\times$$
$$\times\sin\left(\frac{\pi t}{\omega}\right)\cdot\frac{\delta\left(\xi-\xi_1\right)}{l}. \qquad (17)$$

Figure 9, curve 2, presents the flexure caused by action of only one load $q_1(t)$.

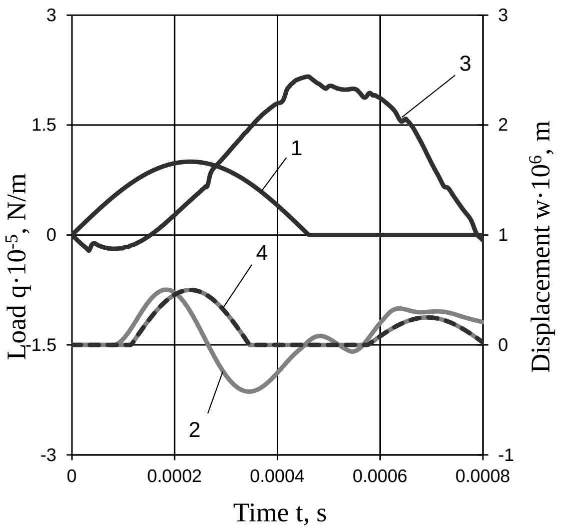


Fig. 9. Rezults of flexure control

Curve 4, which illustrates the controlling criterion, is superimposed with a dotted curve that demonstrates fulfilment of the control criterion due to action of the specified and controlling load, i.e. a curve obtained by implementing the control operation.

Conclusion

In this paper, solutions of two inverse problems in solid mechanics are obtained. The first problem is identification of a motionless and moving concentrated annular non-stationary load acting on a cylindrical shell by applying displacement values in a certain point of the shell; and identification of two motionless concentrated loads. The second problem is controlling vibrations in a certain point of the cylindrical shell by introducing an additional load, which is the controlling one.

The direct problem was solved by using expansions of sought for functions into Fourier series, and the Laplace transform; the inverse problem was solved by using the theory of Volterra integral equations of 1-st kind, and Tikhonov's regularization algorithm.

By applying Tikhonov's regularization algorithm, one can effectively identify unknown non-stationary loads, which is crucial for efficient design of reliable constructions containing cylindrical shells as elements. Its application also allows constructing a theoretical basis for implementing control of different parameters of the deflected mode of a cylindrical shell.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Дві обернені нестаціонарні задачі осесиметричного деформування пружної циліндричної оболонки скінченної довжини

Анотація. Проблема. *Серед багатьох задач механіки деформівного твердого тіла існує цілий клас задач, які відносяться до обернених. В свою чергу серед обернених задач багато задач є некоректно поставленими. Отримання точного аналітичного розв'язку таких задач пов'язано з* *певними труднощами математичного характеру і потребує застосування спеціальних методів.* ***Мета.*** *Метою дослідження є отримання аналітичних розв'язків обернених задач з ідентифікації нестаціонарного навантаження та управлінню нестаціонарними коливаннями циліндричної оболонки з несиметричними граничними умовами.* ***Методологія.*** *При дослідженні була використана уточнена теорія оболонок середньої товщини. Для отримання розв'язку прямої задачі використовувалося розкладання у ряди Фур'є, теорія інтегральних рівнянь та перетворення Лапласа. При розв'язанні обернених задач був використаний метод регуляризації А.М. Тихонова.* ***Результати.*** *В результаті дослідження отримано розв'язки двох обернених задач механіки деформівного твердого тіла. Перша задача – ідентифікація нерухомого та рухомого зосередженого осесиметричного нестаціонарного навантаження, що діє на циліндричну оболонку, на основі значень переміщень в будь-якій точці оболонки; ідентифікація двох нерухомих зосереджених навантажень. Друга задача – управління коливаннями в будь-якій точці циліндричної оболонки за допомогою введення допоміжного зосередженого навантаження. Отримано чисельні результати, що демонструють виконання критерію управління в результаті дії заданого та допоміжного навантаження.* ***Оригінальність.*** *Отримано аналітичні розв'язки обернених задач механіки деформівного твердого тіла для циліндричної оболонки середньої товщини з несиметричними граничними умовами закріплення.* ***Практичне значення.*** *Отримана методика дозволяє ефективно ідентифікувати невідоме нестаціонарне навантаження, що є важливим для раціонального проектування надійних конструкцій, що містять циліндричні оболонки. Її використання дозволяє також побудувати теоретичну базу для реалізації управління параметрами пружнодеформівного стану елементів конструкцій у вигляді циліндричних оболонок.*

Ключові слова: *циліндрична оболонка, нестаціонарне навантаження, обернена задача, регуляризація, ідентифікація, управління коливаннями.*

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