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Determination of the average torsional stiffness of tires of a double vehicle wheel during its interaction with the road surface

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Annotation. Problem. *In the scientific and methodological recommendations of forensic institutions and in the scientific and technical literature there are currently no universal methods for determining the braking parameters of cargo multi-axle vehicles that have wheels with double tires, which affects the results of drawing up the conclusions of the motor-technical expertise. The lack of universal methods is due to the difficulty of determining the actual braking, especially when the tires of dual wheels interact with the road surface.* ***Goal.*** *The goal is justification of the method of determining the average torsional stiffness of tires of a double vehicle wheel during its interaction with the road surface.* ***Methodology.*** *The approaches adopted in the work to achieve the set goal are based on the theoretical foundations of the deformation of elastic elements, which are located parallel to each other.* ***Results.*** *Equations are determined that allow you to calculate the value of the average torsional stiffness of the tire for wheels that have double, triple or quadruple tires.* ***Originality.*** *The results of the research provide a general idea of the effect of the pressure in the tires of a double wheel on the value of its average torsional stiffness.* ***Practical value.*** *The obtained results can be recommended to expert motor technicians when drawing up a conclusion of an expertise or an expert study. Besides, the results of the study can be used in the educational process during the training of specialists in the field of transport or mechanical engineering.*

Key words: *torsional stiffness, tire pressure, tire stiffness, double wheel, twin wheels, parallel springs.*

Introduction

Road safety to a large extent depends on the road conditions of operation of the vehicle and on the tires installed on its wheels, therefore, not only the result of determining the braking efficiency of the vehicle, but also the conclusions of the auto technical expertise in expert practice will depend on the modeling of the "tire-road surface" pair. It is known that thanks to the friction and adhesion of the tire of the vehicle wheel with the surface of the road surface, its stable movement in a given direction is ensured.

The amount of utilized adhesion between the tire and the surface of the road surface depends on many factors, including the speed of rotation of the vehicle wheel, the condition of the road surface, the parameters of the pneumatic tire, etc.

If you compare the tires of passenger cars, trucks and buses, they differ from each other in terms of dimensions, structure, material structure of individual layers of the tire. If we compare the appearance of such tires, passenger car tires have a lower tire profile height and a more complex tread pattern.

Most often, the tires of different vehicles are compared with each other according to the speed index and the load index, which depend on the stiffness properties of the tires, so the determination of the stiffness of the tire is a fundamental parameter for calculating the actual adhesion forces realized between the tire(s) and the road surface.

Analysis of publications

In the theory of the movement of wheeled vehicles, it is customary to characterize the process of interaction of the tire with the surface of the road surface by utilized adhesion, which occurs in the longitudinal and transverse directions relative to the plane of rotation of the vehicle wheel during the deformation of the tire. It is known that deformations of a vehicle wheel tire, which occur under the influence of external forces acting on the vehicle, can generally be divided into radial (normal), circumferential (tangential), angular and lateral deformation.

It should be noted that the indicated deformations of a pneumatic tire of a vehicle wheel in their pure form almost do not occur, practically all of them are in close connection with each other and appear simultaneously during the operation of a loaded tire, as a result of which the corresponding reactions acting in directions opposite to the forces acting on the vehicle. An increase in the number of wheels on the axles of the vehicle, accordingly, changes the ratio between the radial (normal), circumferential (tangential), angular and lateral deformation of the tire, but in the scientific and technical literature [1-12], as their analysis showed, not enough attention has been paid to the issue of this phenomenon for dual tires, so in this paper we will consider the interaction process of a dual vehicle wheel from the point of view of the deformation properties of its pneumatic tires.

Analysis of scientific and technical literature [12-13] showed that there are several models of interaction between a pneumatic tire and a road surface. The main models that characterize the longitudinal deformation of the tire during its braking can be presented in the form of the following diagrams (Fig. 1).

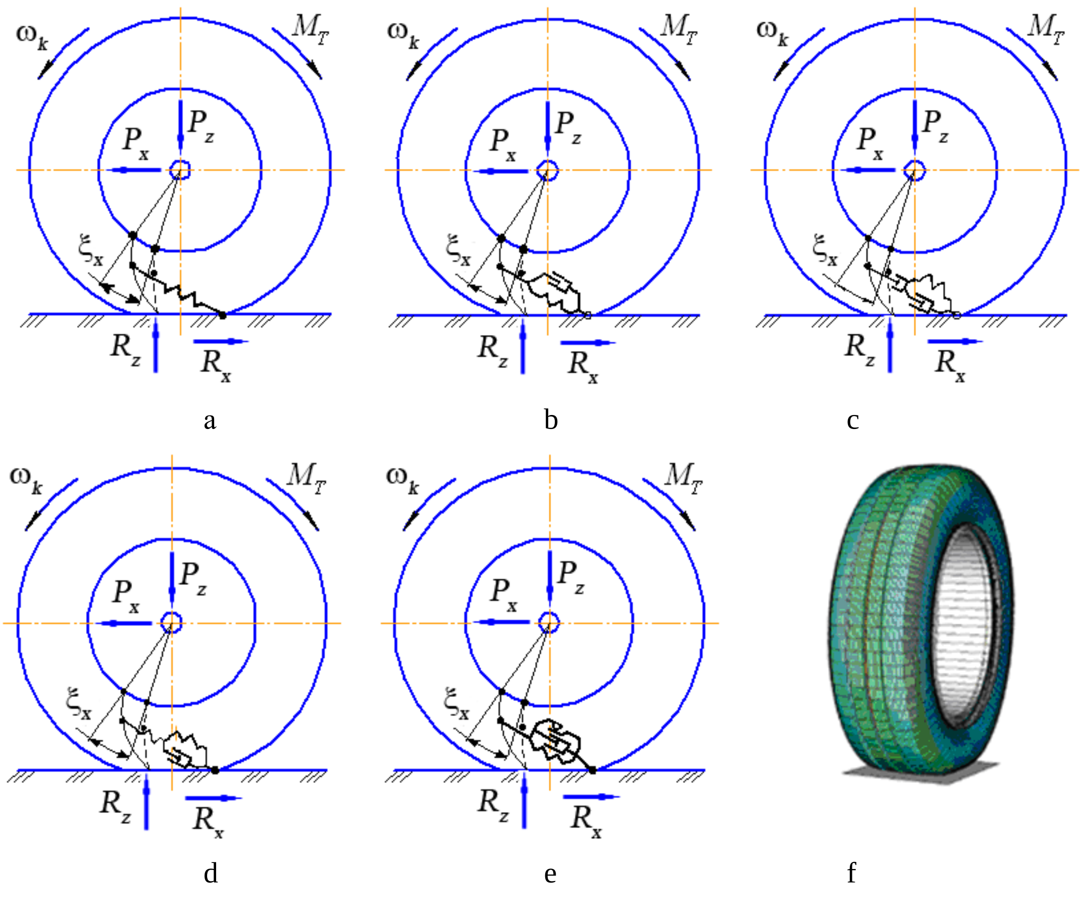

Fig. 1. Schemes of models of interaction of non-rigid pneumatic tires with the road surface: a – with linear stiffness; b – with linear stiffness and viscosity; c – with non-linear stiffness and viscosity; d – with non-linear viscosity and linear stiffness; e – with stiffness, viscosity and dry friction in the tire; f – with the use of finite-element calculation

The analysis of the features of the implementation of mathematical models of dual wheels of a vehicle [14-15] showed that most of them apply only to models of vertical load of tires of dual wheels during the analysis of their influence on the destruction of the road surface. In general, there are no models of the longitudinal interaction of dual-wheel tires with the road surface in the analyzed literature [1-15]. An exception is the paper [16], which considers not the model of the interaction of the tire with the road surface, but the method of determining the braking force between the double-wheel tires and the road surface.

Purpose and Tasks

The purpose of the work is a well-founded method of determining the average torsional stiffness of tires of a double vehicle wheel during its interaction with the road surface.

To achieve the goal, the following tasks must be completed:

- to consider the scheme of circumferential deformation (tangential) deformation of the tire reduced to the plane of the surface of the road surface;
- on the basis of the considered scheme, obtain an equation for determining the average torsional stiffness of tires;
- to consider the possibility of applying the proposed method of determining the average torsional stiffness of dual tires for wheels with a greater number of tires than two;
- carry out simulated modeling of the change in the average torsional stiffness of dual-wheel tires depending on the pressure in them.

Determination of the average torsional rigidity of tires of double automobile wheels

Let's present a mathematical model of dual-wheel tires in the form of two elastic elements (Fig. 2), which is based on the scheme (Fig. 1 a) during the twisting of the tire relative to the surface of the road surface. Figure 2 shows: R_x – the braking force realized between a vehicle wheel with double tires and the road surface, N; R_{x1}^{tire} and R_{x2}^{tire} – correspondingly, the braking force is realized by each tire separately during the interaction of the dual wheel tires with the road surface, H; C_{x1}^{tire} and C_{x2}^{tire} – the torsional stiffness of the corresponding double wheel tires in the longitudinal direction, during its rolling on the surface of the road surface, N·m/rad; y_1, y_2 – distance from the axis of symmetry of the wheel to the axis of symmetry of the corresponding tire, m; y is the distance between the axes of symmetry of the double wheel tires, m; x_1, x_2 – longitudinal movement of the corresponding tires of the double wheel during its braking, m.

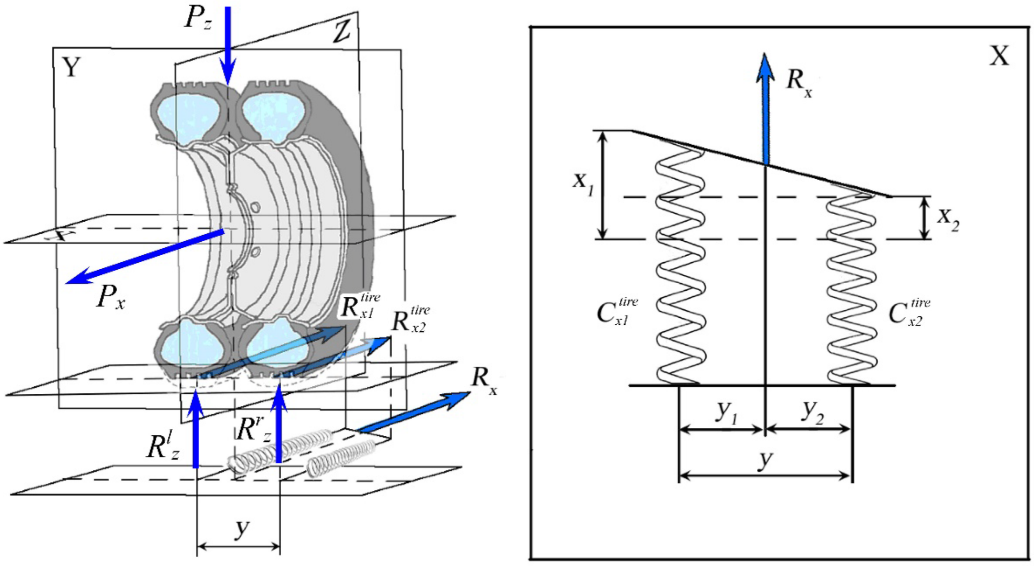

Fig. 2. Diagram of the forces that act during braking of the vehicle on its wheels with double tires

From the similarity of the triangles depicted on the X plane (Fig. 2), it can be seen that the average displacement of the tires of a wheel with double tires is defined as:

$$x = x_2 + \frac{y_2}{y} \cdot (x_1 - x_2) = \frac{y_2}{y} \cdot x_1 + \frac{y_1}{y} \cdot x_2, \qquad (1)$$

and since the braking force realized between a vehicle wheel with double tires and the road surface is equal to the sum of the braking forces realized by each tire separately:

$$R_x = R_{x1}^{tire} + R_{x2}^{tire}, \qquad (2)$$

then you can write from the sum of moments of forces:

$$y_2 \cdot R_{x2}^{tire} = y_1 \cdot R_{x1}^{tire}. \qquad (3)$$

Rewriting equation (3) through the total braking force (2) realized between a vehicle wheel with double tires and the road surface, we get:

$$R_{x1}^{tire} = \frac{y_2 \cdot R_x}{y_1 + y_2} = \frac{y_2 \cdot R_x}{y}, \qquad (4)$$

$$R_{x2}^{tire} = \frac{y_1 \cdot R_x}{y_1 + y_2} = \frac{y_1 \cdot R_x}{y}, \qquad (5)$$

and using Hooke's law, for elastic bodies, we write:

$$x_1 = \frac{R_{x1}^{tire}}{C_{x1}^{tire}} = \frac{y_2 \cdot R_x}{C_{x1}^{tire} \cdot y}, \qquad (6)$$

$$x_2 = \frac{R_{x2}^{tire}}{C_{x2}^{tire}} = \frac{y_1 \cdot R_x}{C_{x2}^{tire} \cdot y}. \qquad (7)$$

By substituting equations (6) and (7) into equation (1), we write the equation (8).

$$x = \frac{R_x}{y^2} \cdot \left(\frac{C_{x2}^{tire} \cdot y_2^2 + C_{x1}^{tire} \cdot y_1^2}{C_{x1}^{tire} \cdot C_{x2}^{tire}} \right), \qquad (8)$$

and given that we will determine $R_x = x \cdot C_{xm}^{tire}$ the total stiffness of tires installed on double wheels of wheeled vehicle according to the equation:

$$C_{xm}^{tire} = \frac{C_{x1}^{tire} \cdot C_{x2}^{tire} \cdot (y_1 + y_2)^2}{C_{x2}^{tire} \cdot y_2^2 + C_{x1}^{tire} \cdot y_1^2}. \qquad (9)$$

In the case when $y_1 = y_2$, i.e. $C_{x1}^{tire} = C_{x2}^{tire}$, equation (9) will take the known form for two parallel springs

$$C_{xm}^{tire} = \frac{4 \cdot C_{x1}^{tire} \cdot C_{x2}^{tire}}{C_{x2}^{tire} + C_{x1}^{tire}} = \sum_{i=1}^{2} C_{xi}^{tire}, \qquad (10)$$

but such a case is possible only when the wheels with double tires are installed on the hub without distortions (Fig. 3), the tires have the same design and wear of the tread pattern, and also the same pressure is set in the tires, otherwise the equation (9) is valid, and not the equation (10).

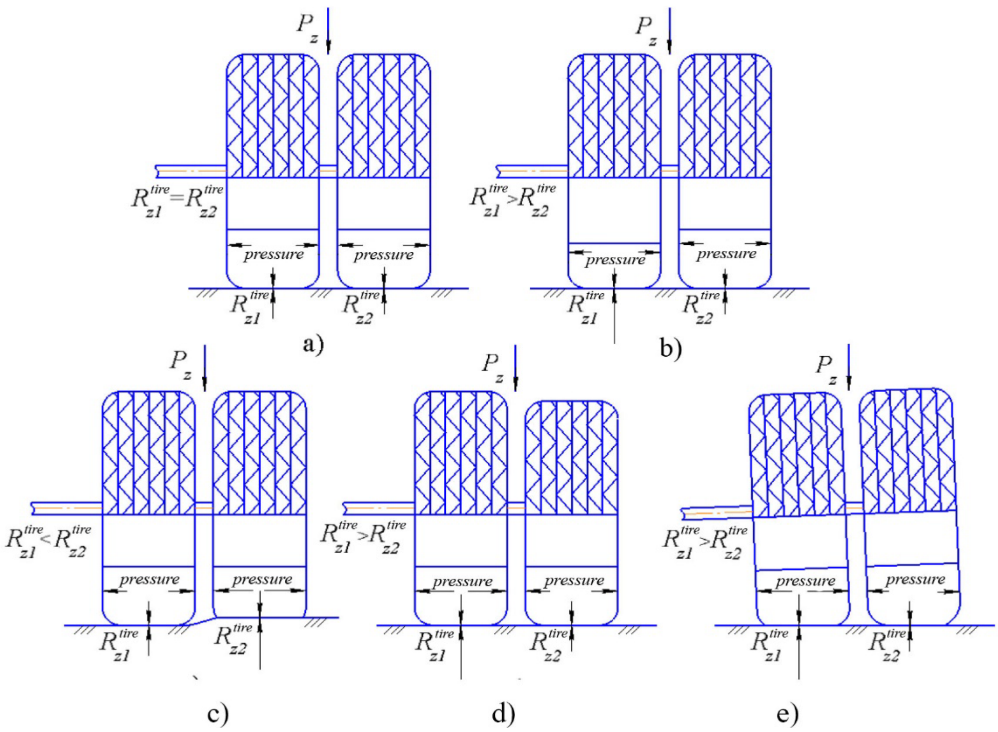

Fig. 3. Schemes of factors that cause unevenness of distances y_1, y_2 from the axis of symmetry of the wheel with double tires to the axis of symmetry of the corresponding tire on the hub: a) idealized ratio $y_1 = y_2$; b) non-uniformity of tire pressures; c) under the condition of unevenness of the surface of the road surface; d) use of different tire diameters; e) subject to axis deformation

In the case of the location of one elastic element to the left and right of the braking force (R_x) realized between the vehicle wheel and the surface of the road surface, we will use the diagram (Fig. 4) of the vertical load of vehicle wheel tires with force to determine the distances. At the same time, the distribution of vertical loads between the tires of one wheel will take place relative to the so-called center of stiffness of the system (c.s.), the shift of which in the case of uneven tire stiffness can be taken into account by the coordinates of the position of the center of stiffness y_1^*, y_2^* and Δy, and relative to the coordinates of the action y_1, y_2 and vertical force $P_z = R_z^l + R_z^r$.

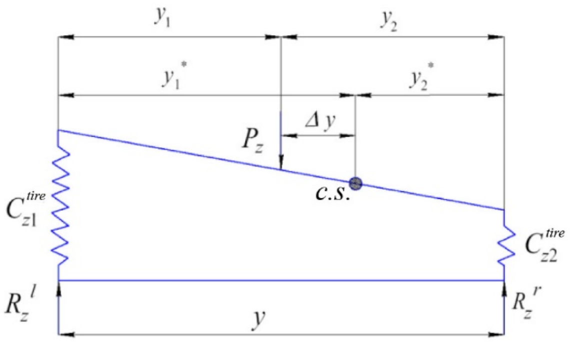

Fig. 4. Load diagram of a wheel with double tires (one tire is located to the left and one to the right of the braking force realized between the vehicle wheel and the road surface)

From the diagram shown in figure 4 and taking into account the methodology proposed in [17], we will write down the coordinates y_1 and y_2 for tires with double tires of the wheel and in the form:

$$y_1 = \frac{R_z^r \cdot (y - \Delta y)}{P_z}, \quad (11)$$

$$y_2 = \frac{R_z^l \cdot (y - \Delta y)}{P_z}, \quad (12)$$

and the coordinates of the position of the center of stiffness of the system y_1^*, y_2^* and Δy are written as:

$$y_2^* = \frac{y_1 \cdot R_z^l}{R_z^r}, \quad (13)$$

$$y_1^* = y - y_2^*, \quad (14)$$

$$\Delta y = \frac{y}{P_z + R_z^l}\left(R_z^l - \frac{P_z C_{z2}^{tire}}{\left(C_{z1}^{tire} + C_{z2}^{tire}\right)}\right), \quad (15)$$

where C_{zi}^{tire} - corresponding normal stiffness of the pneumatic tire, N/m.

Determining the distances y_1 and y_2 for calculating the stiffness of parallel elastic elements, which cause their non-parallel compression or stretching, will be somewhat more complicated in comparison with the case of double arrangement of elastic elements (wheels with double tires), for example, for the cases of the implementation of vehicle tire tires, which are shown in the figure 5 and figure 6.

a b

Fig. 5. Appearance of wheels with quadruple tires: a – wheels with quadruple tires on a Lotus Esprit Turbo passenger car (designed by Jerry Juhan) [18]; b – wheels with four tires on the Big Bud 16V-747 tractor

a

b

Fig. 6. Appearance of cargo vehicles with four-wheel tires: a – semi-trailer with four-wheel tires on the left and right wheels; b – cargo platform KAMAG SHT for loads up to 150 tons [24]

Schemes for the implementation of tires on automobile wheels on such vehicles can have up to four reference points of contact (see Fig. 7 b), which will interact with the surface of the road surface, provided that it is not parallel to the horizon, at different levels from each other.

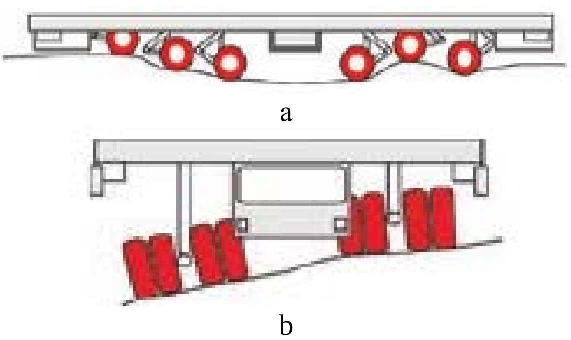

Fig. 7. Conditions of operation of the vehicle [24]: a – along the vehicle; b – across the vehicle

To determine the unknown components y_1 and y_2 in equation (9) in the case of using triple or quadruple tires of the vehicle wheel, we will use the appropriate wheel tire inflation schemes (see Fig. 8, Fig. 9 and Fig. 10) taking into account the redistribution of weight between adjacent pneumatic tires by analogy with the method proposed in [17].

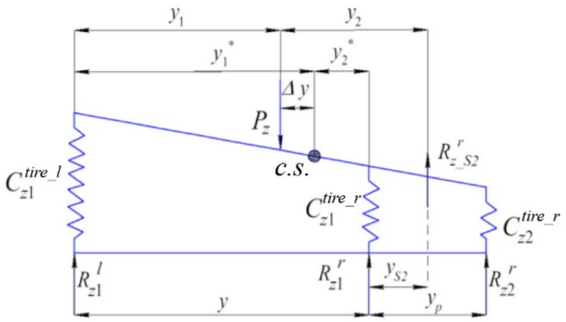

Fig. 8. Scheme with triple tire tires (one tire is located on the left and two on the right of the braking force realized between the vehicle wheel and the road surface)

For the structured tire of the wheel, which has the load scheme shown in Figure 8, the equation for determining the coordinate y_1 and y_2 is written in the form:

$$y_1 = \frac{\left(R_{z1}^r + R_{z2}^r\right)\cdot\left(y - \Delta y\right) + R_{z2}^r \cdot y_p}{P_z}, \quad (16)$$

$$y_2 = \frac{R_{z1}^l \cdot \left(y - \Delta y\right)}{P_z} + \frac{R_{z1}^l \cdot R_{z2}^r \cdot y_p}{P_z \cdot \left(R_{z1}^r + R_{z2}^r\right)}. \quad (17)$$

The coordinate of the position of the center of stiffness of such a system y_2^* is determined by equation (14) by calculating the coordinates y_1^* and Δy by the corresponding equation:

$$y_2^* = \frac{y_1 \cdot R_{z1}^l - R_{z2}^r \cdot y_p}{R_{z1}^r + R_{z2}^r}, \quad (18)$$

$$\Delta y = y_1^* - \frac{y\left(C_{z1}^{tire\_r} + C_{z2}^{tire\_r}\right) + y_p C_{z2}^{tire\_r}}{C_{z1}^{tire\_l} + C_{z1}^{tire\_r} + C_{z2}^{tire\_r}}, \quad (19)$$

where $C_{z1}^{tire\_l}$ and $C_{zi}^{tire\_r}$ – normal stiffness's of the corresponding pneumatic tire of the wheel with triple tires (according to the diagram in figure 8), N/m.

The average torsional stiffness of the tires located to the right of the force P_z is determined from a similar equation (9) in the form (20), which will determine the stiffness of the system shown in figure 8:

$$C_{xm}^{tire\_r} = \frac{C_{x1}^{tire\_r} \cdot C_{x2}^{tire\_r} \cdot y_p^2}{C_{x1}^{tire\_r} \cdot y_{S2}^2 + C_{x2}^{tire\_r} \cdot \left(y_p - y_{S2}\right)^2}. \quad (20)$$

If we accept the assumption that tires with stiffness $C_{x1}^{tire\_r}$ and $C_{x2}^{tire\_r}$ are moved by the same amount, equation (20) can be written in the form (21):

$$C_{xm}^{tire\_r} = \sum_{i=1}^{2} C_{xi}^{tire\_r}. \quad (21)$$

For the structured tire of the wheel, which has the load diagram shown in figure 9, the equation for determining the coordinate y_1 and y_2 is written in the form:

$$y_1 = \frac{R_{z1}^r \cdot \left(y - \Delta y\right)}{P_z} + \frac{R_{z1}^r \cdot R_{z1}^l \cdot y_l}{P_z \cdot \left(R_{z1}^l + R_{z2}^l\right)}, \quad (22)$$

$$y_2 = \frac{\left(R_{z1}^l + R_{z2}^l\right)\cdot\left(y - \Delta y\right) + R_{z1}^l \cdot y_l}{P_z}. \quad (23)$$

The coordinate of the position of the center of stiffness of such a system y_2^* is determined by equation (14) by calculating the coordinates y_1^* and Δy by the equation:

$$y_2^* = \frac{y_1 \cdot \left(R_{z1}^l + R_{z2}^l\right)}{R_{z1}^r}, \quad (24)$$

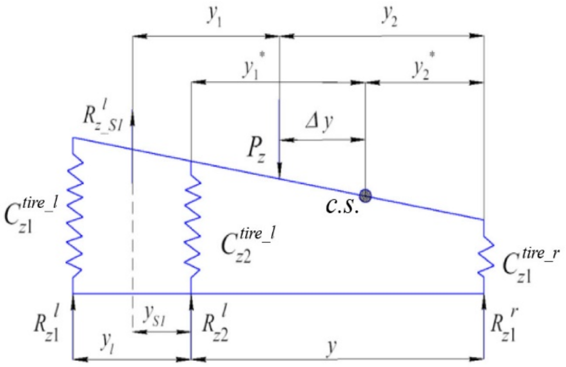

Fig. 9. Scheme with triple tire (one tire is located to the right and two to the left of the braking force realized between the vehicle wheel and the road surface)

$$\Delta y = y_1^* - \frac{y \cdot C_{z1}^{tire\_r} - y_l \cdot C_{z1}^{tire\_l}}{C_{z1}^{tire\_l} + C_{z2}^{tire\_l} + C_{z1}^{tire\_r}}, \qquad (25)$$

where $C_{zi}^{tire\_l}$ and $C_{z1}^{tire\_r}$ – are the normal stiffness's of the corresponding pneumatic tire of the wheel with triple tires (according to the diagram in figure 9), N/m.

The average torsional stiffness of the tires, which determines the stiffness of the system shown in figure 9, is determined from a similar equation (9) in the form:

$$C_{xm}^{tire\_l} = \frac{C_{x1}^{tire\_l} \cdot C_{x2}^{tire\_l} \cdot y_l^2}{C_{x1}^{tire\_l} \cdot y_{S1}^2 + C_{x2}^{tire\_l} \cdot (y_l - y_{S1})^2}, \qquad (26)$$

or if we assume that tires with stiffness $C_{x1}^{tire\_l}$ and $C_{x2}^{tire\_l}$ move by the same amount, we can write equation (26) in the form:

$$C_{xm}^{tire\_l} = \sum_{i=1}^{2} C_{xi}^{tire\_l}. \qquad (27)$$

Thus, it is obvious that for the quadruple wheel tire, which has the load scheme shown in figure 10, the equations for determining the coordinates y_1 and y_2 can be written in the form (28) and (29), respectively:

$$y_1 = \frac{\left(R_{z1}^r + R_{z2}^r\right) \cdot (y - \Delta y) + R_{z2}^r \cdot y_p}{P_z} + \frac{\left(R_{z1}^r + R_{z2}^r\right) \cdot R_{z1}^l \cdot y_l}{P_z \cdot \left(R_{z1}^l + R_{z2}^l\right)}, \qquad (28)$$

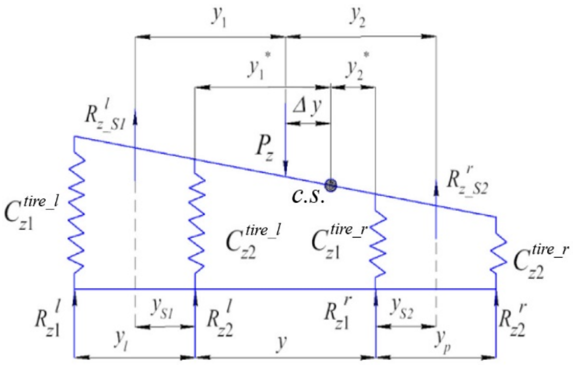

Fig. 10. Scheme with four wheel tires (two tires are located on the left and two on the right of the braking force realized between the vehicle wheel and the road surface)

$$y_2 = \frac{\left(R_{z1}^l + R_{z2}^l\right) \cdot (y - \Delta y) + R_{z1}^l \cdot y_l}{P_z} + \frac{\left(R_{z1}^l + R_{z2}^l\right) \cdot R_{z2}^r \cdot y_p}{P_z \cdot \left(R_{z1}^r + R_{z2}^r\right)}. \qquad (29)$$

The coordinate of the position of the center of stiffness of such a system y_2^* is determined by equation (14) by calculating the coordinates y_1^* and Δy by the equation:

$$y_2^* = \frac{y_1 \cdot \left(R_{z1}^l + R_{z2}^l\right) - R_{z2}^r \cdot y_p}{\left(R_{z1}^r + R_{z2}^r\right)}, \qquad (30)$$

$$\Delta y = y_1^* - \frac{y \cdot \sum_{i=1}^{2} C_{zi}^{tire\_r} + y_p \cdot C_{z2}^{tire\_r} - y_l \cdot C_{z1}^{tire\_l}}{\sum_{i=1}^{2} C_{zi}^{tire\_l} + \sum_{i=1}^{2} C_{zi}^{tire\_r}}, \qquad (31)$$

where $C_{zi}^{tire\_l}$ and $C_{zi}^{tire\_r}$ – are the normal stiffness's of the corresponding pneumatic tire of the wheel with four tires (according to the diagram in Figure 10), N/m.

The average torsional stiffness of the tires, which determine the stiffness of the system shown in Figure 10, is determined from the similar equation (26) and (20), respectively, or by assuming that tires with torsional stiffness $C_{x1}^{tire\_l}$ and $C_{x2}^{tire\_l}$, as well as tires with torsional stiffness $C_{x1}^{tire\_r}$ and $C_{x2}^{tire\_r}$ move by the same value, the average tire stiffness can be determined from equation (27) and (21), respectively, to solve equation (9).

Torsional stiffness of the tire for loads from zero to 40,000 N can be determined by equation (10) [3, 13, 45, 46], which are semi-empirical in nature. This dependence takes into account the effect on torsional stiffness only of the load on the tire and the pressure in the tire and does not take into account other factors:

$$C_x = \left(100 - \frac{B_0}{g \cdot C_x^{P_{\max}} \cdot \exp\left(10 \cdot B_1 \cdot p_{tire}^2\right)}\right) \times \times \left(8,6 \cdot 10^{-2} \cdot R_z - 1,03 \cdot 10^{-6} \cdot R_z^2\right), \quad (32)$$

where p_{tire} – the tire pressure, MPa; g – acceleration of free fall, m/s^2; $C_x^{P_{\max}}$ – the experimental value of the torsional stiffness of the tire at the maximum allowable air pressure in the tire (determined at the maximum allowable load on the tire), Nm/rad; B_0 and B_1 – coefficients determined from equations:

$$B_0 = \frac{1000 \cdot C_x^{P_{\min}}}{\exp\left(B_1 \cdot P_{\min}\right)}; \quad (33)$$

$$B_1 = \frac{\ln\left(\dfrac{C_x^{P_{\max}}}{C_x^{P_{\min}}}\right)}{P_{\max} - P_{\min}}, \quad (34)$$

where $C_x^{P_{\min}}$ – the experimental value of the torsional stiffness of the tire at the minimum permissible air pressure in the tire (determined at the maximum permissible load on the tire), N·m/rad.

Thus, it can be argued that neglecting the features of the distribution of the vertical load between the tires of the wheels of a vehicle that has wheels with double, solid or quadruple tires can significantly affect the magnitude of the utilized adhesion force between the tires of such wheels and the road surface, therefore, during modeling it is necessary to take into account the peculiarities of the change in the torsional stiffness of the tire depending on the ratio y_1 and y_2.

The normal stiffness of a pneumatic tire, in equations (15), (19) – (21), (25) – (27) and (31), as shown by the analysis of scientific and technical literature [20], is not difficult to determine using the well-known equation (35), obtained on the basis of a multifactor bench experimental study:

$$C_z = \frac{\left(a_z \cdot R_{zA}^{n_z}\right)^{\frac{1}{n_z}-1}}{n_z \cdot a_z^{\frac{1}{n_z}}} \quad (35)$$

where $a_z \approx 0.041 p_{tire}^{-1.76}$ – the coefficient of influence of pressure in the tire (p_{tire}) on the normal stiffness of the tire, which is determined in [20]; $n_z \approx 0.45 + 0.38 \cdot p_{tire}$ – the coefficient of influence of pressure in the tire (p_{tire}) on the normal stiffness of the tire, which is determined in [20]. As noted in work [20], the use of equation (35) gives an error in calculations of no more than 17% at pressures in the tire different from the nominal pressures by no more than 25%.

To determine the braking efficiency of freight vehicles that have single and double tires on automobile wheels, we determine the value of the torsional stiffness of the tires installed on the wheels of such vehicles according to equation (9) taking into account equations (11), (12) and (15), and for other cases, taking into account the relevant equations (16), (17), (19), or (22), (23), (25), or (28), (29), (31).

The results of modeling the nature of the change in the torsional stiffness of the tire according to equation (10) show that with an increase in the normal (vertical) reaction (R_z), which acts in the spot of contact of the tire with the road surface, a greater non-linearity of the torsional stiffness is manifested (Fig. 11) at R_z > 20,000 N, than with vertical reactions R_z < 20,000 N.

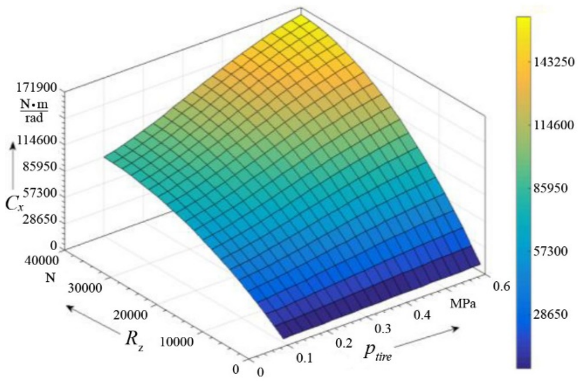

Fig. 11. Modeling the nature of the change in the torsional stiffness of the 11.00 R20 wheel tire with a single tire as a function of the tire pressure and the normal reaction (R_z), which acts in the contact patch of the tire with the road surface

Analyzing the effect of tire pressure on the torsional rigidity of the tire, it can be seen from figure 11 that with an increase in the normal reaction (R_z), which acts in the contact patch of the tire with the road surface, a significant non-linearity when the pressure in the pneumatic tire decreases from 0.6 MPa to 0.1 MPa.

From figure 11, it can also be seen that at tire pressures from 0.4 MPa to 0.6 MPa, the torsional stiffness of a pneumatic tire with a single tire varies in a small range from 2500 N·m/deg to 3000 N·m/deg during the change of normal response (R_z) in the range from 25.000 N to 40.000 N.

Modeling the nature of the change in the normal stiffness of the tire (Fig. 12) according to equation (35) showed that the results of the simulation, in a wide range of pressure changes in the tire and the normal response in the spot of contact of the tire with the road surface, do not contradict the experimental studies given in work [20], therefore, the use of equation (35) is appropriate when determining the distances y, y_1, and y_2 (Fig. 4, 8, and 10) for wheels with double, triple, or quadruple tires.

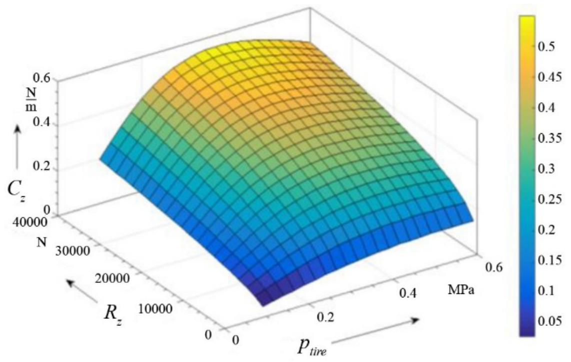

Fig. 12. Modeling the nature of changes in the normal stiffness of a 11.00 R20 wheel tire with a single tire depending on the pressure in it (p_{tire}) and the normal reaction (R_z), which acts in the spot of contact of the tire with the road surface

The results of modeling the nature of the change in torsional stiffness (C_{xm}^{tire}) of the tires of the double wheel 11.00 R20 according to the equation (9), taking into account the change in the normal stiffness (C_{zi}) of each tire according to the equation (32) and taking into account the change of the torsional stiffness (C_{xi}^{tire}) of each tire of the wheel with double tires, show that the torsional stiffness C_{xm}^{tire} of the tires of such a wheel nonlinearly decreases (Fig. 13) by 17% in case of a decrease in pressure in one of the tires of the wheel (left or right) and almost 100% loading of the tire of a double wheel in which the pressure is maintained at a level not lower 0.6 MPa.

In the case of simultaneous pressure reduction in both tires, the torsional stiffness C_{xm}^{tire} will decrease by 33% with an unchanged load on the double vehicle wheel.

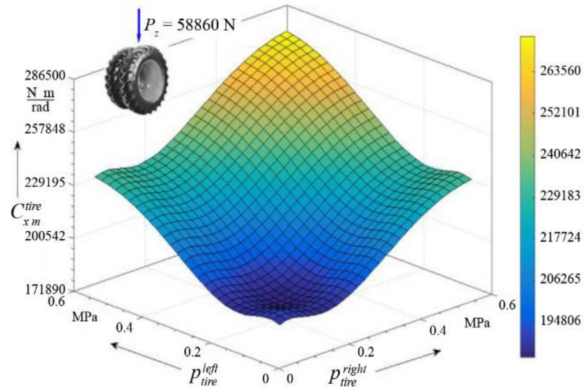

Fig. 13. Modeling the nature of the change in torsional stiffness (C_{xm}^{tire}) of double-wheel tires 11.00 R20 depending on the pressure in the left (p_{tire}^{left}) and right tire (p_{tire}^{right}) of the wheel during normal (vertical) reaction R_z = 58860 N

If the load on a double 11.00 R20 vehicle wheel is reduced, for example by two times (Fig. 14), the nature of the change in torsional rigidity remains the same as with a full normal load (Fig. 13), but the non-linearity of the torsional stiffness is expressed to a greater extent when the pressure in one of the tires of the double wheel (left or right) is reduced. The reduction in torsional stiffness occurs by no more than 14% at almost 100% loading of the dual-wheel tire, in which the pressure is maintained at a level of at least 0.6 MPa.

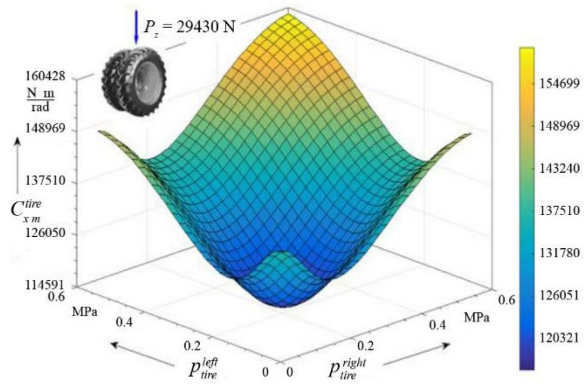

Fig. 14. Modeling the nature of the change in torsional stiffness (C_{xm}^{tire}) of double-wheel tires 11.00 R20 depending on the pressure in the left (p_{tire}^{left}) and right tire (p_{tire}^{right}) of the wheel at normal (vertical) reaction R_z = 29430 N

It should be noted that in the range of pressures in dual-wheel tires from 0.4 MPa to 0.6 MPa, the change in the torsional stiffness of the tire occurs in an insignificant range (Fig. 13 and Fig. 14), as in the case of modeling the torsional stiffness of a vehicle wheel with a single strapping (Fig. 11). Such a change in the torsional stiffness of the 11.00 R20 double wheel tires indicates that the implementation of the traction properties of the tires of such a wheel will take place in a stable range than when the pressure in one of the tires is in the range from 0.15 MPa to 0.35 MPa. The non-linearity of the change in torsional stiffness (C_{xm}^{tire}) of dual-wheel tires in the pressure range from 0.15 MPa to 0.35 MPa is caused primarily by the non-linearity of the change in distances y_1 and y_2 (Fig. 6) and the additional load of the tire (R_z) in which the pressure remains at the level of below 0.6 MPa when the pressure in the other tire drops below 0.35 MPa.

The non-linear change in distances y_1 and y_2, as can be seen from figure 15, depends on the vertical load on the double wheel by no more than 10%, when the pressure in one of the tires drops to a level that is in the pressure range from 0.15 MPa to 0.35 MPa, and by 44% depending on the tire pressure.

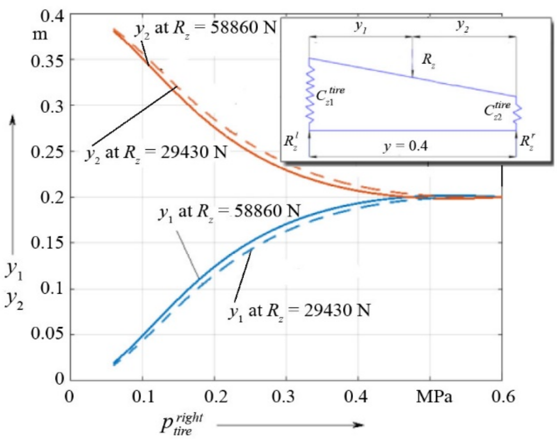

Fig. 15. Modeling of the nature of the change in distances y_1 and y_2 for a double wheel 11.00 R20 depending on the pressure in the right tire (p_{tire}^{right}) of the wheel during normal (vertical) reaction R_z = 29430 N and R_z = 58860 N

When modeling the nature of the change in the distances y_1 and y_2 (Fig. 15), the redistribution of the vertical load between the dual wheel tires occurred according to a linear law, therefore the non-linearity of the distance change is related to the non-linearity of the influence of the pressure in the dual wheel tires on the normal stiffness (C_{zi}).

Conclusion

The performed theoretical study of the nature of the interaction of a double vehicle wheel with the road surface showed that the magnitude of the utilized adhesion force between the tires of such a wheel and the road surface depends on the nature of the change in the average angular stiffness of the tires. If there is a difference between the distances associated with the axis of symmetry of the wheel and the axis of symmetry of the corresponding tire, there is a decrease in the average angular stiffness of tires of wheels that have other than single tires, when the pressure in one of the tires of such a wheel is reduced.

A theoretical study of the nature of the effect of the average torsional stiffness of the tires of wheels that do not have a single tire showed that the value of the average stiffness of the tires of such wheels does not linearly affect the adhesion properties of the "tire-pavement surface" friction pair due to the full implementation of the angle of rotation of the tire relative to the road surface coating.

Analysis of the nature of the change in the torsional stiffness of the tire from the weight parameters of the vehicle showed that an increase in the mass of the vehicle leads to a decrease in the traction properties of its tires, and therefore to a decrease in the amount of deceleration. Reducing the mass of the vehicle, on the contrary, increases the traction properties between the tire and the road surface, which accordingly has a positive effect on the braking efficiency of the vehicle, since its deceleration will increase in proportion to the decrease in the weight of the vehicle.

If there is a difference between the distances associated with the axis of symmetry of the wheel and the axis of symmetry of the corresponding tire, there is a decrease in the average angular stiffness of the tires of wheels that have other than single tires by 14%, when the pressure in one of the tires of such a wheel is reduced by 80%.

A theoretical study of the nature of the influence of the average stiffness of tires of wheels that do not have a single tire showed that the value of the average stiffness of the tires of such wheels does not change when the tire pressure changes in the range from 0.15 MPa to 0.35 MPa. This is caused, first of all, by the non-linearity of the change in distances y_1 and y_2 and the additional load of the tire (R_z) in which the pressure remains at a level not lower than 0.6 MPa when the pressure in the other tire drops to a level not lower than 0.35 MPa.

It was established that the non-linear change in the distances y_1 and y_2 depends on the vertical load on the double wheel by no more than 10%, when the pressure in one of the tires drops to a level that is in the pressure range from 0.15 MPa to 0.35 MPa, and by 44% depending on the tire pressure. In the case of a simultaneous decrease in pressure in both tires, the torsional stiffness will decrease by 33% with an unchanged load on the double vehicle wheel.

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Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Визначення середньої крутильної жорсткості шин здвоєного автомобільного колеса при його взаємодії з поверхнею дорожнього покриття

Анотація. Проблема. *На даний час в науково-методичних рекомендаціях судово-експертних установ та в науково-технічній літературі відсутні універсальні методики щодо визначення параметрів гальмування вантажних багатовісних транспортних засобів, які мають колеса з подвійним ошинуванням, що впливає на результати складання висновків автотехнічної експертизи. Відсутність універсальних методик обумовлена складністю визначення дійсних гальмових, сил особливо при взаємодії шин здвоєних коліс з поверхнею дорожнього покриття.* ***Мета.*** *Обґрунтування методу визначення середньої крутильної жорсткості шин здвоєного автомобільного колеса при його взаємодії з поверхнею дорожнього покриття.* ***Методологія.*** *Прийняті в роботі підходи для досягнення поставленої мети базуються на теоретичних основах деформування пружних елементів, які розташовані паралельно один одному.* ***Результати.*** *Визначено рівняння, які дозволяють розрахувати величину середньої крутильної жорсткості шини для коліс, які мають подвійне, потрійне або четверне ошинування.* ***Оригінальність.*** *Результати проведеного дослідження дають загальне уявлення про вплив тиску в шинах здвоєного колеса на величину його середньої крутильної жорсткості.* ***Практичне значення.*** *Отримані результати можуть бути рекомендовані експертам автотехнікам при складанні висновку експертизи або експертного дослідження, а також результати дослідження можуть бути використані в навчальному процесі при підготовці фахівців галузі транспорт або механічної інженерії.*

Ключові слова: *крутильна жорсткість, тиск в шині, жорсткість шини, подвійне колесо, здвоєне колесо, паралельні пружини.*

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