UDC 629.3.017.5 DOI: 10.30977/AT.2219-8342.2023.52.0.05

To the issue of decomposition of mathematical models of disturbed motion of complex discrete-continuous dynamic systems

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Abstract. Problem. *Usually, complex dynamic systems are characterized by a high order of equations in their mathematical models and intricate interconnections among subsystems that constitute the complex dynamic system. Naturally, the analysis and synthesis of complex dynamic systems require the use of powerful computational systems with vast memory and high processing speed. Goal. The aim is quantitative assessment of the influence of the continuum part of a dynamic system on the behavior of its discrete part and the possibility to perform decomposition of the mathematical model of perturbed motion of a discrete-continuum system based on this assessment.* ***Methodology.*** *The measures are aimed at addressing issues that are based on the discrete-continuum capabilities and their decomposition.* ***Results.*** *For three types of complex dynamic systems described by infinite-dimensional systems of ordinary differential equations, quantitative assessments of the influence of the continuum part on the discrete part of the system have been proposed. These assessments determine the decomposition of the mathematical models of perturbed motion for such systems.* ***Originality.*** *Simplified mathematical models for three types of discrete-continuous dynamic systems were obtained.* ***Practical value.*** *The obtained results can be recommended for the study of the specialized course on the motion characteristics of cargo trucks during the transportation of liquids in tanks. Through optimization using the MATLAB software package, it is possible to simulate various parameters of both the cargo truck and the cargo being transported.*

Key words: *discrete-continual system; complex technical object; decomposition of the mathematical model*

Introduction

Under a dynamic system, we understand an object of any physical nature that evolves over time in the space of its states. A dynamic system is called discrete-continuum if its mathematical model of perturbed motion contains both ordinary differential equations and partial differential equations. A complex dynamic system refers to a system composed of numerous dynamic subsystems that interact, giving rise to new properties that are absent at the subsystem level. Typically, complex dynamic systems are characterized by a high order of equations in their mathematical models and intricate interconnections among subsystems that constitute the complex dynamic system. Naturally, the analysis and synthesis of complex dynamic systems have required the use of powerful computational systems with vast memory and high processing speed. In the 1960s, the development of new weapons systems and military technology provided a powerful impetus for the development of a general theory of complex systems and computational technology, which became an essential attribute in creating complex systems. However, the elementary basis of computational technology at that time, relying on the use of high-inertia vacuum devices (electron tubes), did not allow for the rapid development of complex military objects. Indeed, modeling the stabilization processes of the R-16 rocket, which consists of two stages with two tanks each containing liquid fuel and oxidizer, using the M-20 computer required up to 60 hours of continuous operation. It is understandable that developers of complex technical objects strive to simplify their mathematical models or decompose them by separating the "fast" and "slow" motions of the dynamic system and considering them separately. Advancements in the elementary basis of modern computers have led to a significant increase in their memory capacity and processing speed.

These circumstances have mitigated the problem associated with the "curse of dimensionality" in analyzing and synthesizing complex dynamic systems. Contemporary computational tools are capable of implementing highly complex software products associated with analyzing and synthesizing technical objects whose mathematical models have high dimensionality. However, the problem of decomposing mathematical models of complex dynamic systems remains relevant. Simplified mathematical models of technical objects indeed allow us to understand the physical essence of dynamic processes occurring in such objects and even assess some dynamic characteristics of complex systems manually, without relying on powerful computational tools and software.

Typically, a complex technical object can be represented as two interacting parts. One part of the object, which contains components with concentrated programs, is referred to as the discrete part of the object, while the other part, consisting of components with distributed parameters, is called the continuum part of the object. Overall, a technical object that encompasses interconnected discrete and continuum parts is referred to as discrete-continuum.

Usually, the required dynamic characteristics of a technical object are determined by the behavior of the discrete part. The continuum part of the object introduces perturbations into the behavior of its discrete part.

Analysis of publications

With the emergence of complex technical objects, primarily mobile military objects such as battleships, cruisers, and destroyer escorts with large-caliber main armaments, heavy transport planes and high-capacity bombers, intercontinental ballistic missiles with liquid rocket engines, and spacecraft with dense solar panel arrays, the problem of decomposing the mathematical models of their disturbed motion immediately arose before the designers. The works of A.N. Krylov, S.P. Timoshenko, B.G. Galerkin, and I.G. Bubnov addressed the dynamics of discrete-continuum objects and developed mathematical models for their disturbed motion, as well as approximate engineering methods for analyzing such systems. However, it was only with the advent of electronic digital computers that methods for decomposing mathematical models of discrete-continuum technical objects were developed. In the paper [1,2], the fundamentals of the method of partial oscillators are presented, with which the mathematical model of a discrete-continuum object is reduced to an infinite-dimensional model of a conditional discrete object through its decomposition, which involves considering a limited number of partial oscillators. The basics of the theory of decomposition of dynamic systems, based on the separation of "fast" and "slow" motions, are presented in works [3,4], where the influence of "fast" motions on "slow" motions is discussed, as well as the conditions under which this influence can be avoided. The possibility of controlling "fast" processes in order to influence "slow" processes is considered in the paper [5].

The mentioned works assume that the "main" coordinates of a complex dynamic system, which determine its nature and behavior, are the coordinates of its discrete part, i.e., the coordinates that describe the "small-scale" processes. In such systems, the motion that determines its continuous part consists of "slow" motions of its discrete part, changing at its own pace over time. Rockets with liquid rocket engines fall into this category of objects. The rocket's body with the payload compartment belongs to its discrete part, while the oxidizer in the stages' tanks belongs to its continuous part. During the development of the Ukrainian R-16 rocket (1951-1961), several experimental samples experienced accidents, primarily due to the influence of forced oscillations of the free surfaces of the fuel and oxidizer in the second stage's tanks on the rocket's body motion, resulting in a loss of stability.

There are also discrete-continuous objects in which the motion of the discrete part is considered "fast," while the motion of the continuous part is considered "slow." An example of such objects is a large-sized tanker truck used for transporting fuel with a tank capacity exceeding 20m^3. The large surface area of free fuel in the tank leads to relatively low-frequency oscillations of the fuel, significantly affecting the directional stability of the vehicle.

Purpose and Tasks

The aim of the study is to quantitatively assess the influence of the continuous part of a dynamic system on the behavior of its discrete part and explore the possibility of decomposing the mathematical model of the perturbed motion of a discrete-continuous system based on this assessment.

Mathematical models of perturbed motion of discrete-continuous technical objects and their decomposition.

According to the stated objective, let's consider the mathematical models of the reproduced motion for each of the three types of discrete-continuous capabilities and their decomposition.

Let's examine a stabilized tank gun as a discrete-continuous object. In the works [6,7], a mathematical model of perturbed motion of the object has been developed, which can be expressed as follows:

$$I_n\ddot{\varphi}(t)-\int_r^l m_1(x)\frac{\partial^2 y(x_1 t)}{\partial t^2}dx=M_c(t) \qquad (1)$$

$$\begin{aligned}&m_1(x)\cdot\ddot{\varphi}(t)+m(x)\frac{\partial^2 y(x_1 t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}EI(x)\times\\&\times\frac{\partial^2 y(x_1 t)}{\partial x^2}+\mathfrak{z}\frac{\partial^2}{\partial x^2}EI(x)\frac{\partial^3 y(x_1 t)}{\partial x^2\partial t}=F(x_1 t)\end{aligned} \qquad (2)$$

where $\varphi(t)$ – is the angular misalignment between the undeformed axis of the barrel channel and the aiming line; $y(x,t)$ – is the deviation of the current point on the deformed axis of the barrel channel from the nominally undeformed axis; $M_c(t)$ – is the stabilizing moment generated by the stabilizer; $F(x,t)$ - is the distributed force along the barrel caused by vertical oscillations of the tank hull, where:

$$F(x_1 t)=m(x)\left[\ddot{Z}_k(x)-g\right]$$

$m(x)$ – linear mass of the barrel; $m_1(x)$ - a quantity related to the linear mass by the equation:

$$m_1(x)=m(x)\cdot(x-r)$$

$EI(x)$ – is the flexural stiffness of the barrel; In – is the moment of inertia of the tank gun about the axis of rotation; r – is the distance from the axis of rotation to the point of connection of the elastic part of the barrel with the spring part; l –is the distance from the axis of rotation to the muzzle; $\ddot{Z}_k(t)$ – is the vertical acceleration of the tank hull, $\mathfrak{z}$ – is the coefficient of internal friction of the barrel material; g – is the acceleration due to gravity.

According to the Fourier method, we assume:

$$y(x_1 t)=\sum_{i=1}^{\infty}\gamma_i(x)T_i(t)$$

Then the mathematical model (1, 2) of the discrete-continuous object with consideration of boundary conditions is as follows:

$$y(x_1 t)=\Big|_{x=2}=0;\ \frac{\partial y(x_1 t)}{\partial x}\Big|_{x=2}=0$$

$$EI(x)\frac{\partial^2 y(x_1 t)}{\partial x^2}=\Big|_{x=1}=0;\ \frac{\partial}{\partial x}EI(x)\frac{\partial^2 y(x_1 t)}{\partial x^2}\Big|_{x=1}=0$$

and the conditions of orthogonality of the eigenmodes of elastic vibrations of the barrel $\gamma_i(x)$, $(i=\overline{1,\infty})$:

$$\int_{r_l}^{l} m_1(x)\gamma_j(x)dx=a_j;(j=\overline{1,\infty})$$

$$E\int_r^l\frac{\partial^2}{\partial x^2}\left[I(x)\gamma_i''(x)\gamma_j(x)\right]dx=\begin{cases}0 & at\ i\neq j;\\ b_j & at\ i=j;\end{cases}\quad (j=\overline{1,\infty});$$

$$\int_{r_l}^{l} m(x)\gamma_i(x)\gamma_j(x)dx=\begin{cases}0 & at\ i\neq j;\\ c_j & at\ i=j;\end{cases}\quad (j=\overline{1,\infty});$$

$$\int_r^l F(x,t)\gamma_j(x)dx=f_j(t);\ \ (j=\overline{1,\infty})$$

It is expressed in the form of an equivalent system of ordinary differential equations of infinite order.

$$I_n\ddot{\varphi}(t)-\sum_{i=1}^{\infty}a_i\ddot{T}_i(t)=M_c(t)\,; \qquad (3)$$

$$\begin{aligned}&a_i\ddot{\varphi}(t)+C_i\ddot{T}_i(t)+\mathfrak{z}b_i\dot{T}_i(t)+b_iT_i(t)=\\&=K_i\left[\ddot{Z}_i(t)-g\right];(i=\overline{1,\infty}),\end{aligned} \qquad (4)$$

where the coefficient K_i is determined by the formula

$$K_i=\int_r^l m(x)\cdot\gamma_i(x)dx\,;\ (i=\overline{1,\infty}).$$

The problem of decomposing the model (3), (4) lies in reducing its dimensionality, in other words, excluding from consideration the conditions (4), the solution of which provides a small destabilizing influence on the change in the angle φ(t).

In equation (4), the notation $a_i\ddot{\varphi}(t)$ represents the parameters of perturbations caused by the stabilized motion of the tank's gun relative to the trunnion axis. Let's assume that the parametric perturbations are absent. Such a regime occurs when the stabilizer is turned off and the gun is locked. Then the elastic vibrations of the barrel are described by the differential equations:

$$C_i \ddot{T}_i(t) + \mathfrak{z} b_i \dot{T}_i(t) + b_i T_i(t) = \\ = K_i \left[\ddot{Z}_K(t) - g \right]; (i = \overline{1,\infty}). \qquad (5)$$

Let us show that $T_i(t) = T_i o + \Delta T_i(t)$, where $T_i o$ – is the static component of the solutions of equations (5) determined by the static deflection of the barrel; $\Delta T_i(t)$ – is the dynamic component. Then each of the equations (5) is split into two equations

$$b_i T_{i0} = K_i g \; ; (i = \overline{1,\infty}) \qquad (6)$$

$$C_i \Delta \ddot{T}_i(t) + \mathfrak{z} \; b_i \Delta \dot{T}_i(t) + b_i \Delta T_i(t) = \\ = K_i \ddot{Z}_K(t); \;\; (i = \overline{1,\infty}). \qquad (7)$$

The static deflection of the barrel at the muzzle can be estimated by substituting $i = 1$ into formula (6).

$$T_{i0} = \frac{K_i}{b_1} g \, , \qquad (8)$$

and the time constant of the i-th mode of elastic vibrations of the barrel, according to equation (7), can be estimated by the formula:

$$T_i = \sqrt{\frac{C_i}{b_1}} \, . \qquad (9)$$

The equation (3), which describes the motion of the stabilized tank barrel, is used to estimate the time constant of the barrel. For this purpose, we denote the perturbation from the elastic vibrations of the barrel as

$$M_B(t) = \sum_{i=1}^{\infty} a_i \ddot{T}_i(t) \, , \qquad (10)$$

And write equation (3) in the form:

$$I_n \ddot{\varphi}(t) = M_c(t) = M_B(t) \, . \qquad (11)$$

The stabilizing moment applied to the tank gun is proportional to the pressure difference of the working fluid, ΔP(t), in the chambers of the hydraulic actuator.

$$M_c(t) = K_M \Delta P(t) \, ,$$

which, in turn, is proportional to the rotation angle $\beta(t)$ of the electromagnet armature of the electro-hydraulic amplifier.

$$\Delta P(t) = K_Д \beta(t)$$

As a result, equation (11) takes the following form:

$$I_n \ddot{\varphi}(t) = K_M K_Д \beta(t) + M_B(t) \qquad (12)$$

From the analysis of equation (12), we can write the relationship for the time constant of the tank gun as follows:

$$T_n = \sqrt{\frac{I_n}{K_m K_d}} \qquad (13)$$

In the work [8], numerical values of parameters for a tank gun with a caliber of 125 mm, installed on the modern Ukrainian tank BM "Oplot," are provided. The parameter values for the discrete part of the object described by equation (12) are as follows: $I_n = 736.9\ N{\cdot}m{\cdot}s^2$, $K_m = 0.6\ 10^{-3}\ N{\cdot}m{\cdot}Pa^{-1}$, $K_d = 1.238\ 10^7\ Pa$.

Substituting these values into equation (13) allows us to calculate the time constant of the discrete part of the object, which is $T_n = 0.316s$. Numerical values of parameters for the continuous part of the object for the first three modes of elastic vibrations of the barrel are presented in Table 1.

Table 1 Values of parameters for the continuous part

N тона	a_i $H{\cdot}c^2$	b_i $H{\cdot}м^{-1}$	Ci $H{\cdot}м\text{-}1{\cdot}c^2$	Ki $H{\cdot}м\text{-}1{\cdot}c^2$	T_{ic}, c
1	$9.721{\cdot}10^2$	$2.213{\cdot}10^5$	$2.152{\cdot}10^3$	$3.612{\cdot}10^2$	$0.986{\cdot}10^{-1}$
2	$7.999{\cdot}10^2$	$3.193{\cdot}10^6$	$1.941{\cdot}10^3$	$3.994{\cdot}10^2$	$0.247{\cdot}10^{-1}$
3	$6.34{\cdot}10^2$	$1.786{\cdot}10^7$	$2.144{\cdot}10^3$	$2.286{\cdot}10^2$	$1.096{\cdot}10^{-2}$

In the works [1,2], the following quantitative assessment of the influence of "fast" motions on "slow" motions is proposed: when considering the slow motions, they can be neglected if the ratio of the time constant of the "fast" component, Tiδ, to the time constant of the "slow" component satisfies the inequalities:

$$\frac{T_{i\delta}}{T_M} \le 0,05\,;\ \ (i = 1, \Pi)\,. \tag{14}$$

For the considered object, the ratio (14) is given by:

$$\frac{T_{1c}}{T_n} = 0,312 > 0,05\,;$$
$$\frac{T_{2c}}{T_n} = 0,078 > 0,05\,;$$
$$\frac{T_{3c}}{T_n} = 0,347 < 0,05\,.$$

These equations lead to the conclusion that in the mathematical model (3), (4), it is sufficient to consider the first two types of barrel oscillations for the tank gun.

The accuracy of stabilizing the conditionally undeformed barrel of the lateral gun is 20 angular seconds, while the static deflection of the barrel at the muzzle level according to formula (8) is 16 mm. Therefore, the required accuracy is achieved through two devices that are an integral part of the stabilizer - the tank ballistic computer, which corrects the alignment of the barrel axis, and the firing permission device, which grants permission to fire when the conditionally undeformed axis of the barrel deviates from the aiming line by no more than 20 angular seconds, and also when the deformed axis of the barrel forms a straight-line coinciding with the aiming line.

Study of discrete-continuous dynamical systems

Let us consider the second group of discrete-continuum dynamic systems in which the discrete and continuum parts have close natural frequencies, which can lead to resonance and, consequently, the destruction of the technical object. As mentioned earlier, such objects include intercontinental ballistic missiles (ICBMs) with liquid rocket engines (LREs).

LRE rockets use liquid fuel and oxidizer, so each stage of an ICBM contains two tanks filled with liquid propellant, the oscillations of the free surface of which destabilize the stabilization processes of the missile's body.

In [2], the author presents the developed method of partial oscillators, according to which the discrete-continuum mathematical model of the perturbed motion of the C5M stage of the "Cyclone-3" carrier rocket is formulated in the scattering channel as follows:

$$\begin{gathered}\ddot{\psi}(t) = a'_{\psi\psi}\,\dot{\psi}(t) + a_{\psi\psi}\psi(t) + \\ \sum_{i=1}^{\infty} a''_{\psi\alpha i}\,\ddot{\alpha}_i(t) + \sum_{i=1}^{\infty} a''_{\psi\beta i}\,\ddot{\beta}_i(t) + a_{\psi\delta}\delta(t)\end{gathered} \tag{15}$$

$$\begin{gathered}T_\delta^2\,\ddot{\delta}(t) + 2T_\delta\zeta\,\dot{\delta}(t) + \delta(t) = \\ = K_1\,\psi(t) + K_2\,\dot{\psi}(t)\,;\end{gathered} \tag{16}$$

$$\begin{gathered}\ddot{\alpha}_i(t) + 2\xi_{\alpha i}\omega_{\alpha i}\,\dot{\alpha}_i(t) + \omega_{\alpha i}^2\,\alpha_i(t) = \\ = a''_{\alpha i\psi}\,\ddot{\psi}(t)\,;\ \ (i = \overline{1,\infty})\end{gathered} \tag{17}$$

$$\begin{gathered}\ddot{\beta}_i(t) + 2\xi_{\beta i}\omega_{\beta i}\,\dot{\beta}_i(t) + \omega_{\beta i}^2\,\beta_i(t) = ; \\ = a''_{\beta i\psi}\,\ddot{\psi}(t)\,;\ \ (i = \overline{1,\infty})\end{gathered} \tag{18}$$

where $\psi(t)$ – is the rotation angle of the longitudinal axis of the stage with respect to the orbital plane; $\delta(t)$ – is the rotation angle of the stage's main engine; $\alpha_i(t)$ – is the generalized coordinate of the *i*-th partial oscillator describing the fuel free surface oscillation; $\beta_i(t)$ – is the generalized coordinate of the i-th partial oscillator describing the oxidizer free surface oscillation; $a_{\psi\psi}$, $a'_{\psi\psi}$, $a_{\psi\delta}$ – are time-varying coefficients characterizing the angular motion of the "solidified" stage; $a''_{\psi\alpha i}$, $a''_{\psi\beta i}$ – are coefficients representing the influence of the *i*-th partial oscillators of the fuel and oxidizer on the angular motion of the rocket; $a''_{\alpha i\psi}$, $a''_{\beta i\psi}$ – coefficients of the influence of the rocket's angular motion on the oscillations of the partial oscillators of fuel and oxidizer; T_δ – is the time constant of the steering drive; ζ – is the damping coefficient of the steering drive; K_1, K_2 – are variable parameters of the stabilizer; $\xi_{\alpha i}$, $\xi_{\beta i}$ – are the damping coefficients of the partial oscillators of the fuel and oxidizer; $\omega_{\alpha i}$, $\omega_{\beta i}$ – are the natural frequencies of the partial oscillators of the fuel and oxidizer, which are determined by the relationships:

$$\omega_{\alpha i} = \sqrt{g\lambda_{\alpha i} th(\lambda_{\alpha i} h_\alpha)}\ ;\ (i = \overline{1,\infty}) \qquad (19)$$

$$\omega_{\beta i} = \sqrt{g\lambda_{\beta i} th(\lambda_{\beta i} h_\beta)}\ ;\ (i = \overline{1,\infty}) \qquad (20)$$

where g – acceleration due to gravity; $\lambda_{\alpha i}$, $\lambda_{\beta i}$ – wave numbers of the partial oscillators of the fuel and oxidizer, respectively, where

$$\lambda_{\alpha i} = \frac{\pi(2i-1)}{d_\alpha}\ ;\ \lambda_{\beta i} = \frac{\pi(2i-1)}{d_\beta}\ ;\ (i = \overline{1,\infty}), \qquad (21)$$

where h_α – fuel tank height, h_β – oxidizer tank height; d_α - fuel tank diameter, d_β - oxidizer tank diameter.

To avoid resonance phenomena in the dynamic system described by differential equations (15) - (18), it is necessary to separate the frequencies of the rocket body's natural oscillations in stabilized motion and the natural frequencies of the partial oscillators in the fuel and oxidizer tanks by installing radial partitions that divide the fuel masses into separate compartments. In this case, the wave numbers (21) can be expressed as follows:

$$\lambda_{\alpha i} = n_\alpha \frac{\pi(2i-1)}{d_\alpha}\ ;\ \lambda_{\beta i} = n_\beta \frac{\pi(2i-1)}{d_\beta}\ ;\ (i = \overline{1,\infty}),$$

where n_α – number of radial partitions in the fuel tank, n_β – number of radial partitions in the oxidizer tank.

Figure 1 shows the arrangement of fuel tanks in the C5M stage of the Cyclone-3 carrier rocket. The fuel tanks are shaped like toroids with an external diameter of D=2R=2.66m. The fuel capacity is 1100 kg, and the oxidizer capacity is 1900 kg.

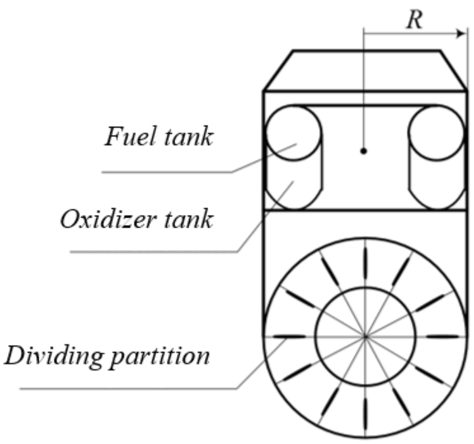


Fig. 1. Schematic diagram of the fuel tank arrangement of the C5M stage of the «Cyclone-3» carrier rocket.

In the absence of additional damping in the tanks (compared to natural damping), the motion of the stage is unstable. Additional damping leads to the emergence of a stability region in the parameter plane of the varying parameters K_1 and K_2 of the stabilizer. This additional damping is achieved by installing twelve radial partitions in the tanks, which increase the damping coefficient and the natural frequency of the liquid oscillations in the tanks.

Let us find the natural frequency of oscillations of the "rigid" stage, whose disturbed motion is described by a system of fourth-order differential equations.

$$\ddot{\psi}(t) = a'_{\psi\psi}\,\dot{\psi}(t) + a_{\psi\psi}\,\psi(t) + a_{\psi\delta}\delta(t) \qquad (22)$$

$$T_\delta^2\,\ddot{\delta}(t) + 2\xi\, T_\delta\,\dot{\delta}(t) + \delta(t) = K_1\,\psi(t) + K_2\,\dot{\psi}(t); \qquad (23)$$

Let us introduce the state vector of the system (22) as follows:

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} \psi(t) \\ \dot{\psi}(t) \\ \delta(t) \\ \dot{\delta}(t) \end{bmatrix}$$

And solve equation (23) with respect to the highest derivative.

$$\ddot{\delta}(t) = -\frac{2\xi}{T_\delta}\,\dot{\delta}(t) - \frac{1}{T_\delta^2}\delta(t) + \frac{K_1}{T_\delta^2}\psi(t) + \frac{K_2}{T_\delta^2}\dot{\psi}(t)\,, \qquad (24)$$

Then the differential equations (22) and (24) can be written in the Cauchy normal form.

$$\begin{aligned} \dot{x}_1(t) &= x_2(t)\,; \\ \dot{x}_2(t) &= a_{\psi\psi}x_1(t) + a'_{\psi\psi}x_2(t) + a_{\psi\delta}x_3(t)\,; \\ \dot{x}_3(t) &= x_4(t)\,; \\ \dot{x}_4(t) &= \frac{K_1}{T_\delta^2}x_1(t) + \frac{K_2}{T_\delta^2}x_2(t) - \frac{1}{T_\delta^2}x_3(t) - \frac{2\xi}{T_\delta}x_4(t)\,, \end{aligned} \qquad (25)$$

We can express the mathematical model of perturbed motion of the "solid" rocket in vector-matrix form

$$\dot{X}(t) = A(K_1K_2)X(t)\,, \tag{26}$$

where the eigenvalue matrix $A(K_1, K_2)$ is expressed as

$$A(K_1K_2)\begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{\psi\psi} & a'_{\psi\psi} & a_{\psi\delta} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_1}{T_\delta^2} & \frac{K_2}{T_\delta^2} & -\frac{1}{T_\delta^2} & -\frac{2\xi}{T_\delta} \end{bmatrix}. \tag{27}$$

The characteristic equation of the vector-matrix differential equation is given by

$$D(K_1K_2S) = \det\left[A(K_1K_2) - ES\right] = 0\,. \tag{28}$$

By substituting matrix (27) into the characteristic equation (28), we obtain

$$\begin{aligned} D(K_1,K_2,S) &= S^4 + \left(\frac{2\xi}{T_\delta} - a'_{\psi\psi}\right)S^3 - \\ &-\left(\frac{2\xi}{T_\delta}a'_{\psi\psi} + a_{\psi\psi} - \frac{1}{T_\delta^2}\right)S^2 - \\ &-\left(\frac{a'_{\psi\psi}}{T_\delta^2} + \frac{2\xi}{T_\delta}a_{\psi\psi}\right)S - \\ &-a_{\psi\delta}\frac{K_2}{T_\delta^2}S - \frac{a_{\psi\psi}}{T_\delta^2} - a_{\psi\delta}\frac{K_1}{T_\delta^2} = 0 \end{aligned} \tag{29}$$

To simplify further calculations, we can rewrite equation (29) as follows

$$\begin{aligned} D(K_1K_2S) &= S^4 + a_1S^3 + a_2S^2 + \\ &+ a_3S + a'_3K_2S + a_4 + a'_3K_1 = 0 \end{aligned} \tag{30}$$

where the coefficients of the equation are determined by the formulas:

$$a_1 = \frac{2\xi}{T_\delta} - a'_{\psi\psi}\,;\; a_2 = -\left(\frac{2\xi}{T_\delta}a'_{\psi\psi} + a_{\psi\psi} - \frac{1}{T_\delta^2}\right)$$

$$a_3 = -\left(\frac{a'_{\psi\psi}}{T_\delta^2} + \frac{2\xi}{T_\delta}a_{\psi\psi}\right);\; a'_3 = -\frac{a_{\psi\delta}}{T_\delta^2}\,;\, a_4 = -\frac{a_{\psi\psi}}{T_\delta^2}\,.$$

In equation (30), we substitute S=α+jω, extract the real and imaginary parts, set them equal to zero, and solve the resulting equations for the varying stabilizer parameters

$$\begin{aligned} K_1(\alpha,\omega) &= \frac{1}{a'_3}\left[3\alpha^4 + 2\alpha^2\omega^2 - \omega^4 + \right. \\ &\left. +2a_1\alpha(\alpha^2+\omega^2) + a_2(\alpha^2+\omega^2) - a_4\right] \\ K_2(\alpha,\omega) &= \frac{1}{a'_3}\left[-4\alpha(\alpha^2-\omega^2) - \right. \\ &\left. -a_1(3\alpha^2-\omega^2) - 2a_2\alpha - a_3\right] \end{aligned} \tag{31}$$

If we set *α = 0* in equations (31), we can use the derived formulas to plot the stability boundary of the system (25) (shown as a shaded line in Figure 2) as α varies from zero to infinity. By increasing α in the negative direction using equations (31), we can construct lines of constant stability degree. At a certain *α = α**, the line of constant stability degree degenerates into the line segment ab. If we choose the stabilizer parameters K_1 and K_2 within the segment ab, the dynamic system (25) has the maximum stability margin. If we choose the parameters at point a ($K_1 = 1.83$, $K_2 = 4.22s$), the roots of the characteristic equation (29) closest to the imaginary axis are purely negative, with $S_1 = \alpha^*$.

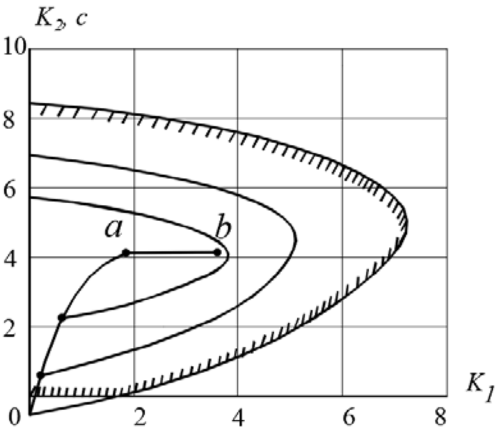


Fig. 2. Lines of equal stability degree of the system.

In the mathematical model (22), (23), let's assume that the rocket's control actuator is inertial.

$$\delta(t) = K_1\psi(t) + K_2\dot{\psi}(t)$$

In this case, the perturbed motion of the "rigid" rocket is described by the differential equation:

$$\ddot{\psi}(t) = \left(a'_{\psi\psi} + a_{\psi\delta}K_2\right)\dot{\psi}(t) + \left(a_{\psi\psi} + a_{\psi\delta}K_1\right)\psi(t)\,. \tag{32}$$

According to equation (32), the square of the natural frequency of oscillations of the "rigid" rocket is equal to:

$$\omega_p^2 = -a_{\psi\psi} - a_{\psi\delta} K_1 \quad (33)$$

The flight of the C5M space stage takes place in very rarefied layers of the atmosphere where the value of the coefficient $a_{\psi\psi}$ is very small. Therefore, the ratio (33) can be simplified and represented as:

$$\omega_p \approx \sqrt{-a_{\psi\delta} K_1} \quad (34)$$

In works [11,12], the values of coefficients for the mathematical model (15)-(18) during the flight of the C5M stage on the active segment of the trajectory, with a duration of 112 seconds, are provided. These values are presented in Table 2.

Table 2. Values of coefficients for the model (15) - (18)

t, s	$a'_{\psi\psi}$, s^{-1}	$a_{\psi\delta}$, s^{-2}	$\omega^2_{\alpha1}$, s^{-2}	$\omega^2_{\beta1}$, s^{-2}	$\omega^2_{\alpha2}$, s^{-2}	$\omega^2_{\beta2}$, s^{-2}	$\omega^2_{\alpha3}$, s^{-2}	$\omega^2_{\beta3}$, s^{-2}
1	2	3	4	5	6	7	8	9
0	-0.0119	-0.643	14.8	14.2	44.4	42.6	74.0	71.0
8	-0.0107	-0.594	14.9	9.94	44.7	29.8	74.5	49.7
16	-0.0110	-0.612	14.9	10.6	44.7	31.8	74.5	53.0
24	-0.0113	-0.626	14.9	11.5	44.7	34.5	74.5	57.5
32	-0.0116	-0.640	14.8	12.2	44.4	36.6	74.0	61.0
40	-0.0116	-0.644	14.7	11.9	44.1	35.7	73.5	59.5
48	-0.0114	-0.640	14.5	11.4	43.5	34.2	72.5	57.0
56	-0.0112	-0.643	14.5	10.6	43.5	31.8	72.5	53.0
64	-0.0131	-0.694	14.9	26.5	44.7	79.5	74.5	132.5
72	-0.0124	-0.721	14.4	31.6	46.2	94.8	77.0	158.0
80	-0.0117	-0.744	13.1	27.1	39.3	81.3	65.5	135.5
88	-0.0094	-0.531	9.1	11.5	28.7	34.5	47.1	57.5
96	-0.0089	-0.505	7.0	10.6	23.7	31.8	39.5	53.0
104	-0.0087	-0.507	8.4	9.6	25.9	28.8	43.2	48.0
112	-0.0080	-0.468	7.72	8.29	23.2	24.9	38.6	41.45

The values of the squared eigenfrequency of rocket oscillations with "solid" propellant, computed at different points of the interval (a, b) shown in Figure 2, and for various moments of the active segment of the flight trajectory, are provided in Table 3.

Table 3. Values of the squared eigenfrequency of the rocket

t, s \ K_1, B	1.75	2.00	2.25	2.50	2.75
1	2	3	4	5	6
0	1.125	1.286	1.447	1.608	1.767
8	1.040	1.189	1.337	1.486	1.634
16	1.071	1.224	1.377	1.530	1.683
24	1.096	1.253	1.410	1.566	1.722
32	1.120	1.280	1.440	1.600	1.760
40	1.127	1.288	1.449	1.610	1.771
48	1.120	1.280	1.440	1.600	1.760
56	1.125	1.286	1.447	1.608	1.767
64	1.215	1.389	1.562	1.736	1.909
72	1.262	1.442	1.623	1.803	1.983
80	1.302	1.488	1.674	1.861	2.045
88	0.929	1.062	1.195	1.328	1.459
96	0.884	1.010	1.137	1.263	1.389
104	0.887	1.014	1.141	1.268	1.393
112	0.819	0.936	1.053	1.170	1.287

The analysis of tables 2 and 3 allows us to conclude that the maximum value of the squared eigenfrequency of the "stiffened" rocket oscillations is reached at 80 seconds of flight, corresponding to the point in figure 2, and it is $\omega^2_{pmax} = 2.045 s^{-2}$. At this moment, the squared eigenfrequencies of the first mode of vibrations of the free surfaces of the propellant and oxidizer are $\omega^2_{\alpha1} = 13.1 s^{-2}$ and $\omega^2_{\beta1} = 27.1 s^{-2}$ respectively. The squared eigenfrequency values corresponding to the second mode are three times higher than those of the first mode, and the values corresponding to the third mode are five times higher. Thus, the first mode of vibrations of the free surfaces of the propellant and oxidizer has a significant influence on the disturbed motion of the rocket, while the higher modes have a lesser effect. Therefore, the infinite-dimensional mathematical model (15)-(18) can be simplified as follows:

$$\ddot{\psi}(t) = a'_{\psi\psi}\,\dot{\psi}(t) + a_{\psi\psi}(t) + a''_{\psi\alpha}\,\ddot{\alpha}(t) + a''_{\psi\beta}\,\ddot{\beta}(t) + a_{\psi\delta}\delta(t);$$

$$T_\delta^2\,\ddot{\delta}(t) + 2T_\delta\xi\,\dot{\delta}(t) + \delta(t) = K_1\,\psi(t) + K_2\,\dot{\psi}(t);$$

$$\ddot{\alpha}(t) + 2\xi_\alpha\omega_\alpha\,\dot{\alpha}(t) + \omega_\alpha^2\,\alpha(t) = a''_{\alpha\psi}\,\ddot{\psi}(t);$$

$$\ddot{\beta}(t)+2\xi_\beta\omega_\beta\dot{\beta}(t)+\omega_\beta^2\beta(t)=a''_{\beta\psi}\ddot{\psi}(t).$$

The third group of discrete-continuous technical objects includes objects in which the motions of the discrete component are considered "fast" and the motions of the continuous component are considered "slow". As an example of such an object, we consider the large-tonnage fuel tanker truck KrAZ-63221 with a 20m3 capacity tank.

In works [14, 15], a mathematical model of the disturbed motion of the object has been developed using the method of partial oscillators. Taking into account the coordinate systems represented in figure 3.

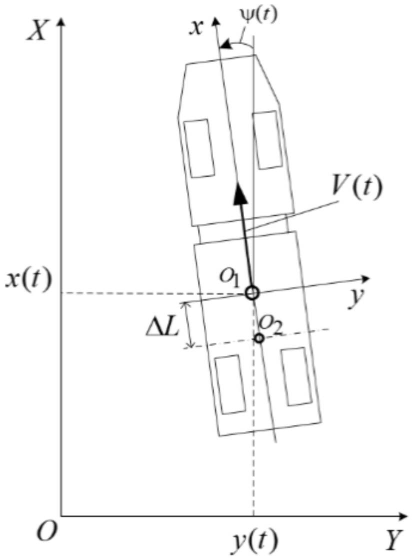


Fig. 3: Coordinate Systems

The model can be expressed as follows:

$$M\dot{V}(t)=-2K_\Gamma P_0(t)-\sum_{K=0}^{\infty}m_K^x\ddot{x}_K(t)-f_cMg; \quad (35)$$

$$\begin{aligned}&I\ddot{\psi}(t)=-0,5BK_\Gamma\Delta P(t)-f_ch_0MV(t)\dot{\psi}(t)+\\&+m_c(t)-\xi\Delta L\sum_{\ell=1}^{\infty}m_\ell^y\ddot{y}_\ell(t)+f_c\sum_{\ell=1}^{\infty}m_\ell^y\ddot{y}_\ell(t)\times\\&\times\left(H_n+h_\ell\right)-f_c\sum_{\ell=1}^{\infty}gm_\ell^y y_\ell(t);\end{aligned} \quad (36)$$

$$\begin{gathered}\ddot{x}_\kappa(t)+\varepsilon_{\kappa x}\dot{x}_\kappa(t)+\omega_{\kappa x}^2x_\kappa(t)=-\dot{V}(t);\\(\mathrm{K}=\overline{1,\infty});\end{gathered} \quad (37)$$

$$\begin{gathered}\ddot{y}_\ell(t)+\varepsilon_{\ell y}\dot{y}_\ell(t)+\omega_{\ell y}^2y_\ell(t)=\\=V(t)\dot{\psi}(t)-\Delta L\ddot{\psi}(t);(\ell=\overline{1,\infty})\end{gathered} \quad (38)$$

where *V(t)* – is the translational velocity of the center of mass; *Ψ(t)* – is the angular deviation of the vehicle's longitudinal axis from the desired direction of motion; *y(t)* – is the lateral deviation of the center of mass from the desired trajectory; $P_0(t)$ – is the pressure of the working fluid at the outlet of the main brake cylinder; *ΔP(t)* – is the pressure difference of the working fluid in the brake lines of the right and left sides of the vehicle; *M* – is the total mass of the vehicle; *I* – is the moment of inertia of the vehicle about its vertical axis; f_c – is the effective coefficient of rolling resistance of all vehicle wheels; *g* – is the acceleration due to gravity; $m_c(t)$ – is the torque of resistance to rotation; *B* – is the track width; h_0 – is the distance from the road surface to the center of mass O_1; K_e – is the proportional coefficient; $x_\kappa(t)$, $y_e(t)$ – are the longitudinal and lateral displacements of the centers of mass of the partial oscillators about the vertical axis of the tank; m_κ^x, m_e^y – are the masses of the partial oscillators, which are determined by the equations:

$$\begin{aligned}m_k^x&=M_{ж}\frac{2th\left(\lambda_k^xh\right)}{\pi^2\lambda_k^xh\left(k-0,5\right)^2};\ (k=\overline{1,\infty}).\\m_\ell^y&=M_{ж}\frac{2th\left(\lambda_\ell^yh\right)}{\pi^2\lambda_\ell^yh\left(\ell-0,5\right)^2};\ (\ell=\overline{1,\infty})\end{aligned} \quad (39)$$

where M_l – liquid mass in the tank; *h* - liquid level in the tank in the absence of oscillations; $\lambda_{\kappa x}$, λ_{ey} – wave numbers of longitudinal and transverse oscillations of the liquid in the tank.

$$\lambda_k^x=\frac{\pi\left(2k-1\right)}{a};\ (k=\overline{1,\infty});$$

$$\lambda_\ell^y=\frac{\pi\left(2\ell-1\right)}{b};\ (\ell=\overline{1,\infty}),$$

where *a*, *b* – length and width of the tank; εκ, ε_e – dissipation coefficients of the partial oscillators:

$$\varepsilon_{kx}=\omega_k\frac{f}{\pi};\ (k=\overline{1,\infty});$$

$$\varepsilon_{\ell y}=\omega_\ell\frac{f}{\pi};\ (\ell=\overline{1,\infty}),$$

where *f* – logarithmic decrement of fuel oscillations; ωκ, ω_e – natural frequencies of the partial oscillators, determined by the formulas:

$$\omega_{kx}^2 = g\lambda_k^x th\left(\lambda_k^x h\right);\ (k=\overline{1,\infty});$$
$$\omega_{\ell y}^2 = g\lambda_\ell^y th\left(\lambda_\ell^y h\right);\ (\ell=\overline{1,\infty}); \qquad (40)$$

where ΔL – distance between the center of mass of the vehicle and the vertical axis of the tank; ξ – force transmission coefficient, determined by the relationship:

$$\xi = \frac{1}{1+\dfrac{4\cdot\sum_{j=1}^{3} A_j^2 n_j}{B^2\cdot\sum_{j=1}^{3} n_j}}$$

where A_j, $(j=\overline{1,3})$ – distance from the center of mass O_1 to the bridges of the vehicle; n_j $(j=1,3)$ – number of wheels on the j-th bridge; H_n – distance from the road surface to the bottom of the tank; he - distance from the bottom of the tank to the center of mass of the *e*-th partial layer, where

$$h_\ell = h - \frac{th\left(\lambda_\ell^y \dfrac{h}{2}\right)}{\lambda_\ell^y};\ (\ell=\overline{1,\infty})$$

The dimensions of the APC-20 tank are: a = 6 m, b = 2. 4m, H = 1.4 m. Figure 4 shows the dependencies of the eigen frequencies of the first three partial oscillators on the fuel level in the tank, obtained using the equations (40). As the fuel level increases, the eigen frequencies increase. The frequency of the third mode of longitudinal oscillations is taken as ω_{3x} = 0.8 Hz, and the frequency of the third mode of lateral oscillations is taken as ω_{3y} = 1.25 Hz.

Figure 5. shows the dependencies of the relative total mass of partial oscillators $\dfrac{\sum_k m_k^x}{M_g}$ *and* $\dfrac{\sum_\ell m_\ell^y}{M_g}$ on the fuel level in the tank, obtained using equations (39).

The analysis of the curves presented in Figure 5 suggests that in the mathematical model of perturbed motion of the fuel tanker vehicle (35) - (38), it is sufficient to set K = 1.3 and e = 1. In this case, the model takes the following form:

$$M\dot V(t) = -2K_\Gamma P_0(t) - m_1^x \ddot x_1(t) - ;$$

$$-m_2^x \ddot x_2(t) - m_3^x \dot x_3(t) - f_c Mg$$

$$I\ddot\psi(t) = -0{,}5BK_\Gamma \Delta P(t) - f_c h_0 MV(t)\dot\psi(t) + m_c(t) -$$
$$-\xi\Delta L m_1^y \ddot y_1(t) + f_c m_1^y \ddot y_1(t)\left(H_n + h_\ell\right) - f_c g m_1^y y(t)$$

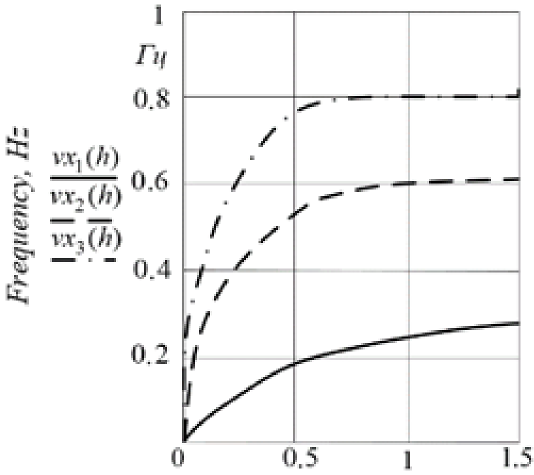
$$\ddot x_1(t) + \varepsilon_{1x}\dot x_1(t) + \omega_{1x}^2 x_1(t) = -\dot V(t)$$

$$\ddot x_2(t) + \varepsilon_{2x}\dot x_2(t) + \omega_{2x}^2 x_2(t) = -\dot V(t)$$

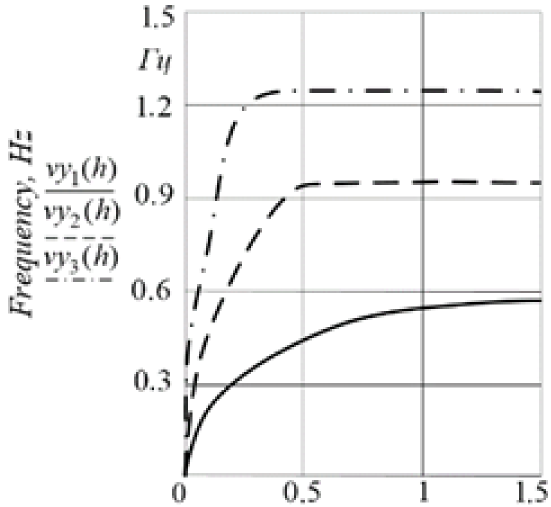
$$\ddot x_3(t) + \varepsilon_{3x}\dot x_3(t) + \omega_{3x}^2 x_3(t) = -\dot V(t)$$

$$\ddot y_1(t) + \varepsilon_{1y}\dot y_1(t) + \omega_{1y}^2 y_1(t) = V(t)\dot\psi(t) - \Delta L\ddot\psi(t)$$

$$\dot y_1(t) = V(t)\psi(t)$$

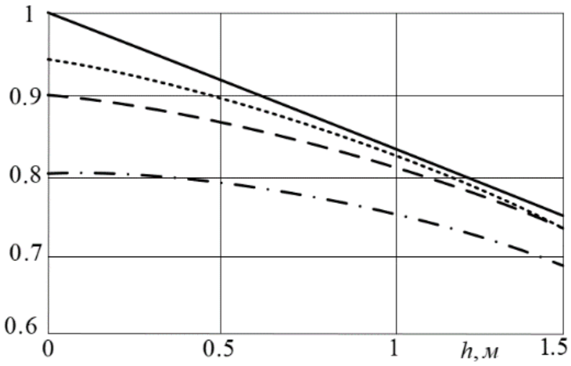


a

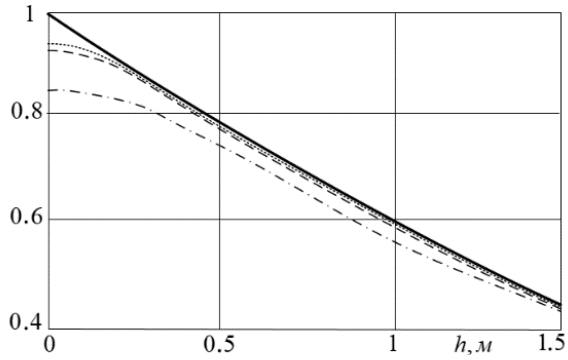


b

Fig. 4. Dependence of partial oscillators' frequencies on the fuel level: a – longitudinal oscillations, b – lateral oscillations.



a



b

Fig. 5. Dependence of the relative mass of partial oscillators on the fuel level: a – longitudinal oscillations, b – transverse oscillations.

Conclusions

For three types of discrete-continuum complex dynamical systems described by infinite-dimensional systems of ordinary differential equations, quantitative estimates of the influence of the continual part of the system on its discrete part are proposed, which determine the decomposition of mathematical models of the perturbed motion of such systems.

Acknowledgement

This work was conducted under the Scientific research "The improvement of the efficiency of operational processes in military automotive technology through the utilization of intelligent systems. ", 04-53-17, funded by the Ministry of Education and Science of Ukraine.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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До питання декомпозиції математичних моделей збуреного руху складних дискретно – континуальних динамічних систем

Анотація. Проблема. *Зазвичай складні динамічні системи відрізняються високим порядком рівнянь їх математичних моделей і складним взаємозв'язком підсистем, що складають складну динамічну систему. Природно, що завдання аналізу та синтезу складних динамічних систем вимагають використання потужних обчислювальних систем із величезною пам'яттю та високою швидкодією.* ***Мета.*** *Кількісна оцінка впливу континуальної частини динамічної системи на поведінку її дискретної частини та можливість на основі цієї оцінки здійснити декомпозицію математичної моделі збуреного руху дискретно-континуальної системи.* ***Методологія.*** *Заходи спрямовані на роботу до вирішення проблем, які ґрунтуються на дискретно-континуальних можливостях та їх декомпозиції.* ***Результати.*** *Для трьох типів дискретно-континуальних складних динамічних систем, що описуються нескінченномірними системами звичайних диференціальних рівнянь, запропоновано кількісні оцінки впливу континуальної частини системи на дискретну частину, що визначають декомпозицію математичних моделей обуреного руху таких систем.* ***Оригінальність.*** *Отримані спрощені математичні моделі для трьох типів дискретно-континуальних динамічних систем.* ***Практична цінність.*** *Отримані результати можна рекомендувати при вивченні дисципліни спеціалізований рухомий склад особливості руху вантажних автомобілів під час транспортування рідин в цистерні. Завдяки оптимізації програмним пакетом MATLAB можливе моделювання з різними параметрами як вантажного автомобіля так і вантажу, що буде перевозитись.*

Ключові слова: *дискретно-континуальна система; складний технічний об'єкт; декомпозиція математичної моделі*

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