UDC 629.3 DOI: 10.30977/AT.2219-8342.2025.56.0.03

Formulation of the problem of topology optimization of automobile and agricultural machinery structures

Tovt B.[1], Leontiev D.[2], Malyi V.[2]

[1]Dnipro State Agrarian and Economic University, Ukraine
[2]Kharkiv National Automobile and Highway University, Ukraine

Annotation. Problem. *The demand for lightweight, durable, and efficient automobile and agricultural machinery structures requires advanced design methods. Topology optimization offers a solution by optimizing material distribution, but its application to complex structures is challenging due to varying loads and constraints. This study focuses on formulating the topology optimization problem to enhance structural efficiency and performance.* ***Purpose.*** *The primary goal of this paper is to contribute to the advancement of the scientific foundation of topology structural optimization, with a particular focus on tackling the complex optimization challenges encountered in automobile and agricultural machinery design.* ***Methodology.*** *Mathematical programming and modeling play a crucial role as foundational tools in the formulation of topology structural optimization problems within the automobile and agricultural machinery industry. Together, these tools facilitate the development of optimized structural designs that are not only lightweight and cost-effective but also capable of withstanding the demanding operational environments of agricultural equipment and automobiles.* ***Results.*** *This paper provides a short review and analysis of the current state of topology structural optimization. It presents both the classical variational formulation and the finite element formulation of the topology optimization problem. The study specifically addresses the problem of minimizing structural mass under stress constraints. The specific highlights are made for formulating the problem of topology optimization of agricultural machinery mechanical structures. The theory is applicable both for agricultural machinery and automobiles.* ***Originality****. This work focuses on the advancement of optimal design theory specifically tailored to address unique challenges in the design of automobile and agricultural machinery structures. To meet these needs, the study develops optimization approaches that integrate the specific mechanical, functional, and economic requirements of automobiles and agricultural equipment.* ***Practical meaning.*** *The practical value of this research lies in its adaptation of existing topology structural optimization problem formulations to address the specific challenges and requirements of the automobile and agricultural machinery industry. This adaptation ensures that the optimization solutions are not only mathematically sound but also practically viable, enabling the design of robust, efficient, and cost-effective heavy machinery components.*

Key words: *topology optimization; agricultural machinery; automobile; FEM; SIMP-method; stress constraints; weight minimum.*

Introduction

Light weight designs are desirable in many industrial applications. Decreasing the structural mass has several immediate results, such as improved performance and reduced fuel consumption which in turn gives reduced emissions and an increased range. A lighter design also gives the possibility to increase payload. Agricultural machinery refers to mechanical equipment and structures used in farming and other agricultural practices.

This category includes a wide range of tools, from simple hand and power tools to complex machines like tractors and the implements they operate or tow. Such machinery plays a vital role in both organic and conventional farming.

Since the rise of mechanized agriculture, these machines have become essential to global food production.

Agricultural machinery is also considered a part of the broader field of agricultural automation, which encompasses more advanced technologies such as digital tools and robotic systems [1]. While agricultural robots are capable of automating all three main stages of farming-diagnosis, decision-making, and execution-traditional motorized equipment mainly automates the execution stage, with the first two stages typically carried out by humans through observation and experience [1].

Analysis of publications

The first steps towards what today is called topology optimization were made in the mid-1960s, when a number of papers on optimization of truss structures were published. The first practical implementation of optimization in the form of point-wise material or voids on a fixed finite element mesh, in order to obtain an optimized shape, was performed by Bendsoe and Kikuchi in 1988. The concept of topology optimization that is common today, i.e., penalization of stiffness for intermediate design variable values in order to achieve a design with only solid material and voids, was introduced by Bendsoe in 1989 and it was later named SIMP, Solid Isotropic Material with Penalization, by Rozvany. The word topology optimization originates from the Greek word *topos* which means landscape or place.

From the very beginning, topology optimization has been synonymous with finding the stiffest topology, given a limited amount of material. Such a problem is formulated as minimizing the compliance $C(x,r) = 1/2\, f(x,r)^T u$; the lower the compliance, the higher the stiffness for the loads $f(x,r)$. This traditional minimum compliance formulation has gained its popularity much because the compliance is a convex function when $K(x)$ depends linearly on x, which makes it computationally efficient.

Purpose and Tasks

The primary goal of this paper is to contribute to the advancement of the scientific foundation of topology structural optimization, with a particular focus on tackling the complex optimization challenges encountered in automobile and agricultural machinery design.

This study aims to develop and refine optimization methodologies that address these specific challenges, ensuring that the resulting structures are not only lightweight and cost-effective but also robust enough to meet the demanding functional and operational requirements of agricultural equipment.

Methodology

Mathematical programming and modeling play a crucial role as foundational tools in the formulation of topology structural optimization problems within the automobile and agricultural machinery industry. These methodologies enable the precise definition and analysis of complex optimization challenges by providing a structured framework for incorporating various design constraints, operational requirements, and performance objectives. In the context of agricultural machinery, mathematical programming helps to account for several specific factors, while modeling provides a detailed representation of the structural behavior under real-world conditions. Together, these tools facilitate the development of optimized structural designs that are not only lightweight and cost-effective but also capable of withstanding the demanding operational environments of agricultural equipment.

Results

This paper provides a short review and analysis of the current state of topology structural optimization. It presents both the classical variational formulation and the finite element (FE) formulation of the topology optimization problem. The study specifically addresses the problem of minimizing structural mass under stress constraints. Several challenges associated with stress-constrained topology optimization are discussed. The specific highlights are made for formulating the problem of topology optimization of agricultural machinery mechanical structures.

Difference in Structural Optimization Problems

Structural optimization is usually divided into three main areas: size, shape and topology optimization. In size and shape optimization, an existing design is parameterized by, usually, a moderate number of design variables, and finding an optimized design often means that the end of the design chain is reached.

That is, the optimized design constitutes the design as it will be manufactured. In size optimization, the design variables can control parameters such as the cross-sectional area of a beam or the thickness of a plate and a fixed FE-mesh is used. In shape optimization, the variables influence the shape of the discretized structure by modifying the shape of the elements, e.g., using pre-defined shapes parameterized using spline functions. Sizing, shape, and topology optimization problems focus on different facets of structural design. In sizing optimization, the objective might involve determining the optimal thickness distribution of a linearly elastic plate or the ideal cross-sectional areas of truss members.

This process seeks to minimize or maximize a physical property – such as mean compliance (external work), peak stress, or deflection – while adhering to equilibrium conditions and constraints on both state and design variables. Here, the design variable represents the plate's thickness, while the state variable could correspond to its deflection. A key characteristic of sizing problems is that the design model's domain and state variables are predetermined and remain fixed throughout the optimization.

The shape optimization aims to determine the optimal configuration of the domain itself, making the domain shape the design variable. Topology optimization, however, extends this concept further by identifying critical features such as the number, location, and shape of voids, as well as the connectivity within the domain.

In contrast to size and shape optimization, topology optimization does not start from a known design and usually does not generate a structural design that is ready for manufacturing; instead, it generates a very first conceptual design that needs to be interpreted into a geometric model before its performance can be evaluated with greater confidence. A design domain is established to define the geometric constraints of the structure, and this domain is discretized using a finite element mesh. Each element within the mesh is assigned a design variable that indicates whether the element contains structural material or represents a void. By adjusting these variables, the structural connectivity evolves to transmit applied loads to the supports in a way that minimizes the objective function, while satisfying the given constraints. This means that the number of design variables in a topology optimization problem is typically very large. Compared to size and shape optimization, topology optimization allows more freedom, as it does not rely on an existing and typically non-optimal design, and therefore topology optimization allows for the greatest gain in performance.

Formulation of problem of Topology Optimization of Agricultural Machinery Structures

The layout problem described below incorporates various aspects of classical structural design optimization. The goal of topology optimization is to determine the optimal structural layout within a defined region. The only known parameters in the problem are the applied loads, potential support locations, the total volume of material available, and possibly additional design constraints, such as the position and size of predefined holes or solid regions. The physical dimensions, shape, and connectivity of the structure are initially unknown. Instead of using standard parametric functions, the topology, shape, and size of the structure are represented by a set of spatially distributed functions defined over a fixed design domain. These functions serve as a parameterization of the continuum's stiffness tensor, and selecting an appropriate parameterization is key to formulating the topology optimization problem correctly [2], [3].

Structural optimization is a general term involving several techniques used to optimize the design of a load carrying structure. The design variables x influence the design of the structure under consideration and these are updated in an automated way such that the objective function $g_0(x)$ is minimized and the constraints are satisfied. A general structural optimization problem can be written in the following form:

$$\min_{x \in R^m} g_0(x),$$

subject to $g_i(x) \le \bar{g}_i$, $i = 1, \dots, c$ and $x_e^l \le x_e \le x_e^u$, $e = 1, \dots, m$, where the c constraints state that the functions $g_i(x)$ need to return a value not greater than the upper limits g_i and the m design variables have the lower and upper bounds x_e^l and x_e^u, respectively.

The functions $g_j(x)$, $j = 0, \dots, c$ can be calculated using the finite element method (FEM) for linearly elastic structures. A nested formulation is used, meaning that the nodal displacement vector $\vec{u} \in R^n$, where n is the number of degrees of freedom of the model, is found as the solution of the state equation:

$$\vec{K}(\vec{x})\vec{u} = \vec{f}(\vec{x}, \vec{r}),$$

where the stiffness matrix $\vec{K}(\vec{x}) \in R^{n \times n}$ is influenced by the design variables and f (x; r) 2 Rn$\vec{f}(\vec{x}, \vec{r}) \in R^n$ is the force vector, which may be influenced by the design variables and which may also be controlled by some external vector $\vec{r}$. This means that the stiffness matrix needs to be invertible, so that the displacements become implicit functions of $\vec{x}$, i.e., $\vec{u} = \vec{u}(\vec{x}) = \vec{K}(\vec{x})^{-1}\vec{f}(\vec{x},\vec{r})$.

Minimum compliance design

In general, the optimal shape design is formulated as a material distribution problem. The set-up is analogous to the formulations for sizing problems for discrete and continuum structures [2], [3]. It is important to note that such problem type has a large scale from a computational point of view, both in state and in the design variables. That's why, the first problems treated in this area usually employed the simplest type of design problem formulation in terms of objective and constraint, namely designing for minimum compliance (maximum global stiffness) under simple resource constraints. As mentioned in [2], this is also conceptually a natural starting point for this exposition as its solution reflects many of the fundamental issues in the field. Based on discussion above, the formulation of problem of topology optimization (Fig. 1) of agricultural machinery structures is proposed to be presented in the similar way. Consider a mechanical element as a body occupying a domain Ω^{mat} which is a part of a larger reference domain Ω in R^2 or R^2.

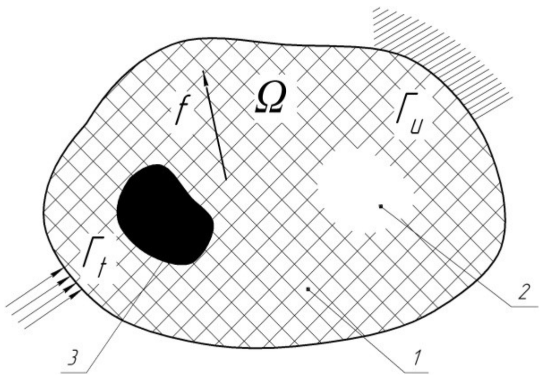

Fig. 1. The generalized shape design problem in the formulation as the optimal material distribution search within a two-dimensional domain: 1 – design point; 2 – point without material; 3 – point with fixed material

The reference domain Ω is chosen so as to allow for a definition of the applied loads and boundary conditions and the reference domain is sometimes called the ground structure, in parallel with terminology used in truss topology design. Referring to the reference domain Ω can be defined as the optimal design problem as the problem of finding the optimal choice of stiffness tensor $E_{ijkl}(x)$ which is a variable over the domain.

Introducing the energy bilinear form (i.e., the internal virtual work of an elastic body at the equilibrium u and for an arbitrary virtual displacement v:

$$a(u,v) = \int_{\Omega} E_{ijkl}(x)\varepsilon_{ij}(u)\varepsilon_{kl}(v)\, d\Omega,$$

where linearized strains $\varepsilon_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ and the load linear form:

$$l(u) = \int_{\Omega} fu\, d\Omega + \int_{\Gamma_T} tu\, ds,$$

the minimum compliance (maximum global stiffness) problem takes the form:

$$\min_{u\in U,E} l(u), \qquad (1)$$

such that $a_E(u,v) = l(v)$ for all $v \in U$, $E \in E_{ad}$. The equilibrium equation is expressed in its weak, variational form, where U represents the space of kinematically admissible displacement fields. The body forces are denoted by f, and t denotes the boundary tractions applied on the traction boundary $\Gamma_T \subset \Gamma \equiv \partial\Omega$. The subscript E indicates that the bilinear form a_E depends on the design variables, specifically through the stiffness distribution.

In equation (1), E_{ad} denotes the set of admissible stiffness tensors relevant to the design problem. In the context of topology optimization, E_{ad} may, for instance, consist of all stiffness tensors that match the properties of a prescribed isotropic material within the (unknown) material region Ω^{mat} , and zero stiffness elsewhere. A resource constraint is then imposed via the volume condition $\int_{\Omega^{mat}} 1 d\Omega \leq V$. The different possible definitions of E_{ad} will be discussed in the next section.

To solve problems of the type described by equation (1) using computational methods, a common approach-adopted throughout this monograph - is to discretize the problem using the finite element method. It is important to recognize that two primary fields are involved in equation (1): the displacement field u and the stiffness distribution E. When the same finite element mesh is used for both fields, and the stiffness E is assumed to be constant within each element, the discrete form of equation (1) can be written as follows:

$$\min_{u,E_e} \vec{f}^T\vec{u}, \qquad (2)$$

such that $K(E_e)\vec{u} = \vec{f}$ for $E_e \in E_{ad}$, where $\vec{u}$ is the displacement vector, $\vec{f}$ is the load vector.

The stiffness matrix K depends on the stiffness E_e in element e, numbered as $e = 1, \dots, N$, and K can be written in the form:

$$K = \sum_{e=1}^{N} K_e(E_e),$$

where K_e is the (global level) element stiffness matrix.

Design parametrization

In structural topology design, the objective is to determine the optimal spatial distribution of a given isotropic material-that is, to identify which regions in the design domain should contain material and which should remain void (i.e., without material). This leads to a geometric representation of the structure analogous to a black-and-white image, where "black" indicates material presence and "white" represents void. In the discrete setting, this corresponds to a black-and-white raster representation, with "pixels" (in 2D) or "voxels" (in 3D) defined by the finite element mesh [2], [3].

Restricting our design to a fixed reference domain Ω, the task becomes finding the optimal material subset $\Omega^{mat} \subset \Omega$. For the optimization problem involving an isotropic material as described above, this formulation implies that the admissible set E_{ad} of stiffness tensors consists of those tensors that satisfy the following condition:

$$E_{ijkl} = 1_{\Omega^{mat}} E^0_{ijkl}, \qquad (3)$$

where $1_{\Omega^{mat}} = \begin{cases} 1 \; if \; x \in \Omega^{mat} \\ 0 \; if \; x \in \Omega / \Omega^{mat} \end{cases}$,

and $\int_{\Omega} 1_{\Omega^{mat}} \, d\Omega = Vol(\Omega^{mat}) \leq V$.

The final inequality represents a constraint on the available material volume, denoted by V, ensuring that the minimum compliance design is achieved within a fixed material budget. The tensor E^0_{ijkl} refers to the stiffness tensor of the prescribed isotropic material. The notation $E_{ijkl} \in L^{\infty}(\Omega)$ is used to specify the appropriate function space for the stiffness distribution in the domain.

This definition of the admissible set E_{ad} results in a spatially distributed, discrete-valued design problem-essentially a 0-1 optimization problem where each point is either material or void.

A widely adopted approach to solving this inherently discrete problem is to relax the binary design variables into continuous ones and introduce a penalization strategy to drive the solution toward discrete (0-1) values. In this framework, the design problem over the fixed domain is reformulated as a sizing optimization problem, where the stiffness matrix is made to depend continuously on a design function interpreted as a material density field. This density function becomes the central design variable.

The objective is to obtain designs that consist almost entirely of regions of full material or complete void, effectively minimizing the presence of intermediate density values. These intermediate values are penalized to mimic the behavior of traditional continuous approximations to 0-1 optimization problems.

One particularly effective and widely used approach is the penalized, proportional stiffness model, known as the SIMP model (Solid Isotropic Material with Penalization):

$$E_{ijkl}(x) = \rho(x)^p E^0_{ijkl}, \qquad (4)$$

where $p > 1$, $\int_{\Omega} \rho(x) \, d\Omega \leq V$, $0 \leq \rho(x) \leq 1$, $x \in \Omega$.

In this context, the "density" $\rho(x)$ serves as the design function, and E^0_{ijkl} represents the stiffness tensor of a specified isotropic material. The term "density" is used because the total volume of the structure is evaluated as $\int_{\Omega} \rho(x) \, d\Omega$, with $\rho(x) \in [0,1]$ indicating the proportion of material present at each point in the domain.

The density function $\rho(x)$ is used to interpolate between void (with material properties equal to zero) and solid material (with full material properties given by E^0_{ijkl}:

$$E_{ijkl}(\rho = 0) = 0,$$
$$E_{ijkl}(\rho = 1) = E^0_{ijkl},$$

This means that if the final design yields density values of either zero or one at every point in the domain, it corresponds to a black-and-white design in which the performance has been evaluated using a physically accurate model. In the SIMP approach, a penalization exponent $p > 1$ is chosen so that intermediate density values are disfavored-i.e., they result in significantly lower stiffness relative to their material cost (volume contribution). In other words, selecting a penalization factor $p > 1$ makes intermediate densities "uneconomical" in the optimization, thus discouraging their presence in the final design.

This penalization effect is achieved implicitly, without the need for an explicit penalty term. For problems where the volume constraint is active, empirical evidence shows that choosing a sufficiently large value of p (typically $p > 3$) leads to designs that are nearly binary (0-1). Moreover, it has been proven in the discrete minimum compliance problem that for sufficiently large ppp, a globally optimal solution with 0-1 characteristics exists-provided that the volume constraint allows for such a design [4], [5].

The SIMP interpolation scheme forms the foundation for most of the computational results presented in the first part of this monograph. The original 0-1 optimization problem is defined over a fixed reference domain, and with SIMP interpolation, the topology optimization problem effectively becomes a standard sizing problem posed on a fixed domain.

The physical interpretability of the SIMP model has frequently been a subject of debate-specifically, whether the interpolation used in SIMP can correspond to a real material, such as a composite, that exhibits the same behavior. It is important to emphasize that the comparison between interpolation schemes like SIMP and micromechanical models is primarily valuable for gaining insight into the nature and implications of such computational methods.

When a numerical scheme successfully produces black-and-white (0-1) designs, the physical meaning of intermediate ("grey") densities becomes less critical and can, to some extent, be disregarded. However, the issue of physical relevance remains pertinent, particularly because many computational methods based on interpolation often result in designs that still contain intermediate density regions. Furthermore, the ability to physically realize all feasible designs becomes especially important when interpreting the results of an optimization process that has been prematurely terminated before full convergence to a 0-1 design. In such cases, understanding the physical implications of intermediate density values is essential for evaluating the reliability and manufacturability of the resulting design.

Implementation of the optimality criteria method

The essential components of the optimality criteria method for implementing the material distribution approach in topology design have been outlined above. These include the fundamental parametrization of the design through the relationship between design variables and stiffness via an appropriate interpolation scheme, as well as the update procedure for the density values based on optimality conditions. This update relies on solving the equilibrium equations, which is typically carried out using the finite element method.

The direct topology design method, employing the material distribution approach, involves numerically determining the globally optimal material density distribution ρρ, which serves as the design variable. When an interpolation scheme that effectively penalizes intermediate densities is used, the method aims to produce a 0-1 (black-and-white) design as the final outcome.

Thus, the optimality criteria method for identifying the optimal topology of a structure made from a single isotropic material comprises the following steps:

Pre-processing of geometry and loading

- Select an appropriate reference domain (often referred to as the ground structure) that accommodates the definition of surface tractions, fixed boundary conditions, and other relevant boundary data.
- Identify the regions within the reference domain that are subject to optimization, distinguishing them from areas that should remain either entirely solid or void throughout the design process.
- Generate a finite element mesh over the ground structure. The mesh should be sufficiently refined to provide an adequate resolution for a bitmap-like representation of the structure. Additionally, it must support the specification of predefined solid or void regions by allowing fixed values of the design variables in those areas. Importantly, this mesh remains fixed throughout the optimization process.
- Define finite element spaces for both the displacement field and the design variable field, treating them as independent fields within the optimization formulation.

Optimization

Compute the optimal distribution of the design variable ρ over the reference domain. The optimization is carried out using displacement-based finite element analysis combined with an optimality criteria update scheme for the material density. The structure of the algorithm is as follows: initialize the design, typically by assigning a homogeneous material distribution across the designable domain.

The iterative part of the algorithm then proceeds:

- Finite Element Analysis: for the current density distribution, compute the displacement and strain fields using the finite element method.
- Evaluate Compliance: calculate the compliance of the current design. If the improvement in compliance compared to the previous iteration is negligible, terminate the optimization. For more precise analysis, continue iterating until the necessary optimality conditions are satisfied.

- Update the Density Field: update the design variable ρ based on the optimality criteria. This step includes solving an inner iteration loop to determine the Lagrange multiplier λ that enforces the volume constraint.
- Repeat the iteration loop until convergence.

In cases where certain parts of the structure are predefined as either solid or void, the update of the design variables should be restricted only to the regions designated for redesign or reinforcement.

Post-processing of results

The optimal material distribution obtained from the algorithm can be interpreted as defining a structural shape-potentially suitable for conversion into a CAD representation. As part of the method, it is essential at the outset to select an appropriate interpolation scheme, such as the SIMP method. Notably, when SIMP is used with a sufficiently high penalization exponent pp, the resulting designs tend to be well-defined, consisting predominantly of regions with either full material or void, with minimal presence of intermediate material density (i.e., minimal "grey" areas). It is important to emphasize that the described algorithm can be implemented on any type of finite element mesh and for any reference domain Ω (i.e., ground structure). This provides the method with considerable flexibility in terms of specifying boundary conditions, loading scenarios, and non-design regions of the structure. However, in practice, rectangular domains (in 2D) or box-shaped domains (in 3D), along with regular meshes composed of squares or cubes, are often used. These choices simplify the implementation and can significantly accelerate the computational performance of the analysis.

On programming complexity

The procedure outlined above does not require substantial programming effort to solve the compliance-based topology optimization problem. For instance, in the case of a rectangular design domain discretized using square finite elements, with Q4 interpolation for displacements and element-wise constant material densities, a complete implementation-including finite element analysis and visualization of the resulting design-can be accomplished in the well-known 99-line MATLAB code.

This compact implementation even incorporates a filtering technique to mitigate common issues such as checkerboarding and mesh dependency, which are inherent to the topology optimization problem. An overview of the computational flow for structural topology design using the material distribution method is illustrated in Figure 2.

Stress constraints

Topology optimization is traditionally used as a tool for finding optimal load paths with respect to stiffness [5], [6]. However, from an engineering point-of-view, few applications have maximum stiffness as the main design criterion. Usually, the design needs to have sufficient stiffness, so that buckling is avoided or the eigenfrequency is above some critical value, and the main design criteria include stress and fatigue requirements and mass minimization. Incorporation of stress constraints in the topology optimization problems is an extremely import topic [7], [8]. However, several challenges must be overcome in order to solve the problem efficiently. Solving of stress constrained continuum topology design problems is covered in [9], [10].

For the 0-1 formulation of the topology design problem a stress constraint is well-defined, but when a material of intermediate density is introduced, the form of the stress constraint is not a priori given [11], [12].

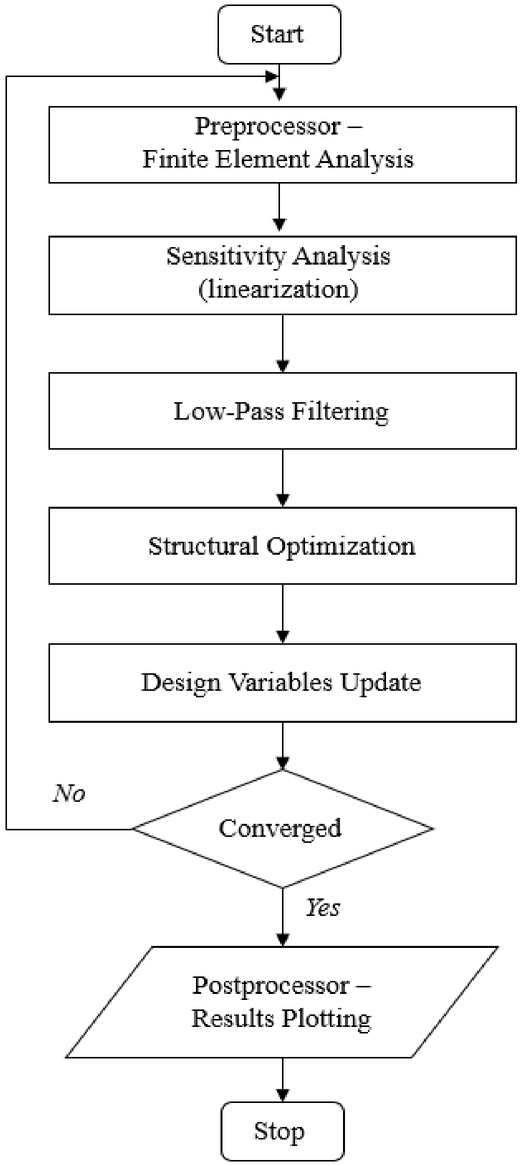

Fig. 2. Computational flow of the structural topology optimization

A stress criterion for the SIMP model should maintain simplicity-similar to the stiffness-density interpolation-and preserve the isotropic nature of the material's stiffness properties within the stress formulation as well [13], [14]. For the model to remain physically meaningful, it is also reasonable that the stress constraint reflects principles consistent with microstructural behavior. This consideration leads to the adoption of a stress constraint in the SIMP framework (with penalization exponent p) based on the von Mises equivalent stress σ_{vM} [15], [16]:

$$\sigma_{vM} \leq \rho^p \sigma_l, \rho > 0. \quad (5)$$

This constraint accounts for the reduction in strength characteristic of a porous medium, where average (macroscopic) stresses are distributed across the local microstructure. As a consequence, "local" stresses can remain finite and non-zero even as the material density approaches zero. This behavior effectively reduces the allowable stress domain by a factor of ρ^p. Notably, the same penalization exponent pp is typically used for both the stiffness interpolation and the stress constraint. Choosing a different exponent is inconsistent with the underlying physical interpretation and, in particular, using an exponent smaller than pp may lead to non-physical outcomes, such as the artificial removal of material, as discussed in [17], [18].

The classical formulation of the stress-constrained topology optimization problem [19], [20] aims to find the minimum-weight structure that satisfies the stress constraints and remains in elastic equilibrium under the applied external loads. This leads to a design problem of the form:

$$\underset{\rho}{min} \sum_{e=1}^{N} v_e \rho_e , \vec{K}\vec{u} = \vec{f}, (\sigma_e)_{vM} \leq \rho_e^p \sigma_l \quad (6)$$

if $\rho > 0, \quad 0 < \rho_{min} \leq \rho_e \leq 1, \quad e = 1, \dots, N,$ where, for instance, the stress is evaluated at the central node of each finite element.

Conclusions

The article examines the key stages in the development of the theory of structural topology optimization, presenting both the classical variational and finite element formulations of the topology optimization problem. It discusses the concept and specific features of implementing the SIMP method for solving such problems.

The formulation of the topology optimization problem is presented as the minimization of the structure's mass while considering stress constraints. A range of challenges associated with introducing these constraints into the optimization problem is addressed. Methods for incorporating stress constraints into topology optimization problems are also analyzed.

Thus, the review and analysis of the current state of the theory of structural topology optimization conducted in this article demonstrate that this scientific field is both relevant and rapidly evolving. For this reason, employing such a modern design tool as topology optimization to address the challenges of creating and improving mechanical structures for agricultural machinery is considered a pressing issue.

The article proposes a formulation of the topology optimization problem for mechanical structures in agricultural machinery, taking into account complex strength constraints, including criteria for allowable stresses.

Conflict of interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

References

1. The State of Food and Agriculture (2022) Leveraging agricultural automation for transforming agrifood systems. Rome: *Food and Agriculture Organization of the United Nations* (FAO). 2022. doi:10.4060/cb9479en. ISBN 978-92-5-136043-9.
2. Haftka, R. T., & Gürdal, Z. (2012). Elements of structural optimization (Vol. 11). *Springer Science & Business Media*. https://doi.org/10.1007/978-94-011-2550-5
3. Kayabekir, A.E., Bekdaş, G., Nigdeli, S.M. (2021). Developments on Metaheuristic-Based Optimization in Structural Engineering. In: Nigdeli, S.M., Bekdaş, G., Kayabekir, A.E., Yucel, M. (eds) *Advances in Structural Engineering—Optimization. Studies in Systems, Decision and Control*, vol 326. Springer, Cham. https://doi.org/10.1007/978-3-030-61848-3_1
4. Holmberg, E., Torstenfelt, B. & Klarbring, A. Stress constrained topology optimization. *Struct Multidisc Optim* 48, 33–47 (2013). https://doi.org/10.1007/s00158-012-0880-7
5. Polajnar, M., Kosel, F. & Drazumeric, R. Structural optimization using global stress-deviation objective function via the level-set method. *Struct Multidisc Optim* 55, 91–104 (2017). https://doi.org/10.1007/s00158-016-1475-5
6. Giraldo-Londoño, O., Russ, J.B., Aguiló, M.A. et al. (2022). Limiting the first principal stress in topology optimization: a local and consistent approach. *Struct Multidisc Optim* 65, 254 https://doi.org/10.1007/s00158-022-03320-y
7. Dixiong, Y., Hongliang L., Weisheng Z., Shi L. (2018). Stress-constrained topology optimization based on maximum stress measures. *Computers &*

Structures, Vol. 198, 23-39 https://doi.org/10.1016/j.compstruc.2018.01.008
8. Amstutz, Samuel & Novotny, A.A. (2010). Topological optimization of structures subject to Von Mises stress constraints. *Structural and Multidisciplinary Optimization.* 41, 407-420. https://doi.org/10.1007/s00158-009-0425-x
9. Guo, Xu & Zhang, Weisheng & Wang, Michael & Wei, Peng. (2011). Stress-related topology optimization via level set approach. *Computer Methods in Applied Mechanics and Engineering* - COMPUT METHOD APPL MECH ENG. 200, 3439-3452. https://doi.org/10.1016/j.cma.2011.08.016
10. Kocvara, Michal & Stingl, Michael. (2012). Solving stress constrained problems in topology and material optimization. *Structural and Multidisciplinary Optimization.* 46, 1-15. https://doi.org/10.1007/s00158-012-0762-z
11. Zhai, X., Chen, F. & Wu, J. (2021). Alternating optimization of design and stress for stress-constrained topology optimization. *Struct Multidisc Optim* 64, 2323–2342 https://doi.org/10.1007/s00158-021-02985-1
12. Le, Chau & Norato, Julian & Bruns, Tyler & Ha, Christopher & Tortorelli, Daniel. (2010). Stress-based topology optimization for continua. *Structural and Multidisciplinary Optimization.* 41. 605-620. https://doi.org/10.1007/s00158-009-0440-y
13. París, José & Navarrina, F. & Colominas, Ignasi & Casteleiro, M. (2010). Block aggregation of stress constraints in topology optimization of structures. *Advances in Engineering Software.* 41. 433-441. https://doi.org/10.1016/j.advengsoft.2009.03.006
14. E. Holmberg, B. Torstenfelt, A. Klarbring (2013). Stress constrained topology optimization. *Structural and Multidisciplinary Optimization*, 48(1):33-47. https://doi.org/10.1007/s00158-012-0880-7
15. R. Picelli, S. Townsend, C. Brampton, J. Norato, H.A. Kim (2018). Stress-based shape and topology optimization with the level set method. *Computer Methods in Applied Mechanics and Engineering*, Vol. 329, 1-23, https://doi.org/10.1016/j.cma.2017.09.001
16. de Leon, D. M., Alexandersen, J., Jun, J. S., & Sigmund, O. (2015). Stress-constrained topology optimization for compliant mechanism design. *Structural and Multidisciplinary Optimization*, 52(5), 929-943. https://doi.org/10.1007/s00158-015-1279-z
17. De Leon, D.M., Gonçalves, J.F. & de Souza, C.E. (2020). Stress-based topology optimization of compliant mechanisms design using geometrical and material nonlinearities. *Struct Multidisc Optim* 62, 231–248 https://doi.org/10.1007/s00158-019-02484-4
18. G. A. Silva, A. T. Beck, O. Sigmund (2019). Stress-constrained topology optimization considering uniform manufacturing uncertainties. *Computer Methods in Applied Mechanics and Engineering*, 344, 512-537. https://doi.org/10.1016/j.cma.2018.10.020
19. Bruggi, Matteo & Duysinx, Pierre. (2012). Topology optimization for minimum weight with compliance and stress constraints. *Structural and Multidisciplinary Optimization.* 46, 369–384. https://doi.org/10.1007/s00158-012-0759-7
20. Collet, M., Bruggi, M. & Duysinx, P. (2017). Topology optimization for minimum weight with compliance and simplified nominal stress constraints for fatigue resistance. *Struct Multidisc Optim* 55, 839–855 https://doi.org/10.1007/s00158-016-1510-6

Tovt Bohdan [1] PhD, Associate Professor of the Department of Higher Mathematics, Physics and General Engineering,
e-mail: btovtua@gmail.com ,
Phone: +38(068) 680-79-31,
ORCID: https://orcid.org/0009-0000-7670-8898
Leontiev Dmytro [2] Doctor of Technical Sciences, Professor of the Department of Automobiles named after A.B. Gredeskul,
e-mail: dima.a3alij@gmail.com ,
Phone: +38(095) 903-68-88,
ORCID: https://orcid.org/0000-0003-4255-6317
Malyi Viktor [2] PhD Student of the Department of Automobiles named after A.B. Gredeskul,
e-mail: victormaly@outlook.com ,
Phone: +38(093) 452-95-50,
ORCID: https://orcid.org/0009-0002-7800-9163

[1]Dnipro State Agrarian and Economic University, Serhii Efremov str., 25, Dnipro, Ukraine, 49009
[2]Kharkiv National Automobile and Highway University, Yaroslava Mudrogo str., 25, Kharkiv, Ukraine, 61002

Постановка задачі топологічної оптимізації конструкцій автомобілів та сільськогосподарської техніки

Анотація. Проблема. *Попит на легкі, міцні та ефективні конструкції автомобілів та сільськогосподарської техніки вимагає застосування сучасних методів проектування. Топологічна оптимізація конструкцій пропонує новий підхід в проектуванні за рахунок оптимізації розподілу матеріалу, проте її застосування до складних конструкцій є непростим завданням через варіативність навантажень та обмежень. Це дослідження зосереджено на формулюванні задачі топологічної оптимізації з метою підвищення ефективності конструкцій.* ***Мета.*** *Основною метою цієї роботи є внесок у розвиток наукових основ топологічної структурної оптимізації з акцентом на вирішення оптимізаційних задач, що виникають у проектуванні автомобілів та сільськогосподарської техніки.* ***Методика.*** *Математичне програмування та моделювання відіграють ключову роль як основні інструменти у формулюванні задач топологічної структурної оптимізації в галузі автомобілебудування сільськогосподарської техніки. У поєднанні ці інструменти сприяють розробці оптимізованих конструкцій, які є легкими, економічно вигідними та здатними витримувати складні умови експлуатації сільськогосподарського обладнання та автомобілів.* ***Результати.*** *У статті представлено короткий огляд і аналіз сучасного* *стану топологічної структурної оптимізації. Розглядаються як класичне варіаційне формулювання, так і формулювання задачі топологічної оптимізації у вигляді скінченно-елементного підходу. Особлива увага приділяється задачі мінімізації маси конструкції під напруженнями. Особливий акцент зроблено на формулюванні задачі топологічної оптимізації механічних конструкцій сільськогосподарської техніки. Теорія придатна для проектування як сільськогосподарської техніки так і автомобілів.* ***Наукова новизна.*** *Дослідження зосереджено на розвитку теорії оптимального проектування, спеціально адаптованої для вирішення унікальних викликів у проектуванні конструкцій автомобілів та сільськогосподарської техніки. Для задоволення вимог у дослідженні розроблено підходи до оптимізації, які інтегрують специфічні механічні, функціональні та економічні вимоги до автомобілів та сільськогосподарського обладнання.* ***Практична значимість.*** *Практична цінність цього дослідження полягає в адаптації існуючих формулювань задач топологічної структурної оптимізації до специфічних викликів і вимог автомобілів та сільськогосподарської техніки. Ця адаптація гарантує, що оптимізаційні рішення будуть не лише математично обґрунтованими, але й практично застосовними, дозволяючи створювати міцні, ефективні та економічно вигідні компоненти висконавантаженої техніки.*

Ключові слова: *топологічна оптимізація; сільськогосподарська техніка; автомобіль, МСК; SIMP; обмеження за напруженням; мінімум ваги.*

Товт Богдан Миколайович[1] к.т.н., доцент кафедри вищої математики, фізики та загальноінженерних дисциплін,
e-mail: btovtua@gmail.com ,
тел.: +38(068) 680-79-31,
ORCID: https://orcid.org/0009-0000-7670-8898

Леонтьєв Дмитро Миколайович[2]**,** д.т.н., професор кафедри автомобілів ім. А.Б. Гредескула,
e-mail: dima.a3alij@gmail.com ,
тел.: +38(095) 903-68-88,
ORCID: https://orcid.org/0000-0003-4255-6317

Малий Віктор Миколайович[2]**,** здобувач кафедри автомобілів ім. А.Б. Гредескула,
e-mail: victormaly@outlook.com ,
тел.: +38(093) 452-95-50,
ORCID: https://orcid.org/0009-0002-7800-9163

[1]Дніпровський державний аграрно-економічний університет, вул. Сергія Єфремова, 25, м. Дніпро, Україна, 49009
[2]Харківський національний автомобільно-дорожній університет, вул. Ярослава Мудрого, 25, м. Харків, Україна, 61002